1. Find an isometry from a part of the xy-plane to the cone

$$\mathbf{x}(u,v) = (v\cos u, v\sin u, v)$$

where $u > 0$ and $0 < v < 2\pi$.

2. We have a chart into the helicoid given by $\mathbf{x}(u,v) = (u\cos v, u\sin v, v)$. We have a chart into the catenoid given by $\mathbf{y}(u,v) = (\cosh u\cos v, \cosh u\sin v, u)$. In these charts we restrict $0 < v < 2\pi$.

Show that the map taking $\mathbf{y}(u,v)$ to $\mathbf{x}(\sinh u, v)$ is an isometry. This shows that the catenoid is locally isometric to the helicoid.

3. Define charts $\mathbf{x}$ and $\mathbf{y}$ as in the previous problem For each t, define

$$\mathbf{x}^t = \cos t\mathbf{y}(u, v+t) + \sin t\mathbf{x}(\sinh u, v+t-\pi/2)$$

Show that $\mathbf{x}^0 = \mathbf{y}$, $\mathbf{x}^{\pi/2} = \mathbf{y}$ and that for each t, show that the map taking $\mathbf{x}(u,v)$ to $\mathbf{x}^t(u,v)$ is an isometry.

For extra credit, use Maple to generate an animation showing this deformation (by isometries) from the catenoid to the helicoid.

4. Let V be a vector space and let A and B be symmetric bilinear forms on V such that $A(v,v) = B(v,v)$ for all $v \in V$. Show that $A = B$.

Hint: Show that $A(v,w)$ can be written in terms an expression only involving terms of the form $A(z,z)$ for certain vectors z.