**Proposition 4.30:** For all $k, m \in \mathbb{N}$, where $m \geq 2$,

$$f_{m+k} = f_{m-1}f_k + f_m f_{k+1}.$$

*Proof*. Your proof goes here. □

An integer $n$ is **odd** if there exists an integer $j$ such that $n = 2j + 1$.

**Proposition 9.A:** Every integer is either even or odd, and no integer is both.

*Proof*. Your proof goes here. Use only material from Chapter 2 or earlier in your proof. □

**Proposition 6.5:** Assume we are given an equivalence relation on a set $A$. For all $a_1, a_2 \in A$, either $[a_1] = [a_2]$ or $[a_1] \cap [a_2] = \emptyset$.

*Proof.* Your proof goes here. □

**Proposition 6.6** (Partial)**:** Let $A$ be a set and let $\Pi$ be a partition of $A$. We define $a \sim b$ if there exists $P \in \Pi$ such that $a \in P$ and $b \in P$. Then $\sim$ is an equivalence relation.

*Proof.* Your proof goes here. □

**Project 6.7:** For each of the following relations defined on $\mathbb{Z}$, determine whether it is an equivalence relation. If it is, determine its equivalence classes.

1. $x \sim y$ if $x < y$.
2. $x \sim y$ if $x \leq y$.
3. $x \sim y$ if $|x| = |y|$.
4. $x \sim y$ if $x \neq y$.
5. $x \sim y$ if $xy > 0$.
6. $x \sim y$ if $x \mid y$ or $y \mid x$.

**Proposition 6.17:** Let $m \in \mathbb{Z}$. Then $m$ is even if and only if $m^2$ is even.

*Proof.* Your proof goes here. □

**Proposition 6.25:** If $a \equiv a' \pmod{n}$ and $b \equiv b' \pmod{n}$ then

$$a + b \equiv a' + b' \pmod{n}$$

and

$$ab \equiv a'b' \pmod{n}.$$

*Proof.* Your proof goes here. □

**Project 6.27:** Study the set $n$ such that $\mathbb{Z}_n$ satisfies the cancellation property (Axiom 1.5). You should form a conjecture, and then prove it. Start working on the problem now, but it won't be due until the next assignment.