Proposition 1.16: If m and n are even integers, then so is $m+n$.

Proof. Let m and n be even integers. To show that $m+n$ is even we must show that there exists an integer j such that

$$m + n = j \cdot 2. \tag{1}$$

Since m and n are even, there are integers p and q such that

$$m = p \cdot 2 \qquad \text{and} \qquad n = q \cdot 2.$$

So

$$\begin{aligned} m + n &= p \cdot 2 + q \cdot 2 \\ &= (p + q) \cdot 2 \end{aligned}$$

by distributivity. Hence equation (1) is satisfied with $j = p + q$, and $m + n$ is even. □

Proposition 1.17(ii) (Our version): If m is an integer and m is divisible by 0, then $m = 0$.

Proof. Suppose that m is an integer that is divisible by 0. So there is an integer j such that

$$m = j \cdot 0.$$

Proposition 1.14 implies $j \cdot 0 = 0$. Hence $m = 0$. □

Proposition 1.18: Let $x \in \mathbb{Z}$. If x has the property that for all $m \in \mathbb{Z}$, $mx = m$, then $x = 1$.

Proof. Suppose x is an integer such that $mx = m$ for all $m \in \mathbb{Z}$. In particular, this equation is satisfied with $m = 1$ and hence

$$1 \cdot x = 1.$$

Proposition 1.7 implies $1 \cdot x = x$, so $x = 1$. □

Proposition 1.19: Let $x \in \mathbb{Z}$. If x has the property that for some nonzero $m \in \mathbb{Z}$, $mx = m$, then $x = 1$.

Proof. Suppose x is an integer such that for some nonzero $m \in \mathbb{Z}$, $mx = m$. From Axiom 1.3 we know that $m = m \cdot 1$. Hence

$$m \cdot x = m \cdot 1.$$

Since $m \neq 0$, we conclude from Axiom 1.5 that $x = 1$. □

Proposition 1.24: Let $x \in \mathbb{Z}$. If $x \cdot x = x$ then $x = 0$ or $x = 1$.

Proof. Suppose x is an integer such that $x \cdot x = x$. To show that either $x = 0$ or $x = 1$, it is enough to show that if $x \neq 0$, then $x = 1$. So suppose $x \neq 0$. Then

$$\begin{aligned} x \cdot x &= x \\ &= x \cdot 1 \end{aligned}$$

by Axiom 1.3. Since $x \neq 0$, Axiom 1.5 implies

$$x = 1.$$

□

Proposition HW2.1: If $m, n \in \mathbb{Z}$ and $m + n = 0$ then $n = -m$.

Proof. Suppose $m, n \in \mathbb{Z}$ and $m+n = 0$. From Axiom 1.4 we know that $m+(-m) = 0$. But then

$$m + n = m + (-m)$$

and Proposition 1.9 implies $n = -m$. □

Proposition 1.25(i): For all $m, n \in \mathbb{Z}$

$$-(m + n) = (-m) + (-n).$$

Proof. Let $m, n \in \mathbb{Z}$. Then

$$\begin{aligned} (m + n) + [(-m) + (-n)] &= [(m + n) + (-m)] + (-n) && \text{by additive associativity} \\ &= [m + (n + (-m))] + (-n) && \text{by additive associativity} \\ &= [m + ((-m) + n)] + (-n) && \text{by additive commutativity} \\ &= [(m + (-m)) + n] + (-n) && \text{by additive associativity} \\ &= [0 + n] + (-n) && \text{by Axiom 1.4} \\ &= n + (-n) && \text{by Proposition 1.8} \\ &= 0 && \text{by Axiom 1.4.} \end{aligned}$$

Proposition HW2.1 then implies

$$(-m) + (-n) = -(m + n).$$

□

Proposition 1.25(ii): For all $m \in \mathbb{Z}$

$$-m = (-1) \cdot m.$$

Proof. Let $m \in \mathbb{Z}$. Then

$$\begin{aligned} m + (-1) \cdot m &= 1 \cdot m + (-1) \cdot m && \text{by Proposition 1.7} \\ &= (1 + (-1)) \cdot m && \text{by Proposition 1.6} \\ &= 0 \cdot m && \text{by Axiom 1.4} \\ &= 0 && \text{by Proposition 1.14.} \end{aligned}$$

Proposition HW2.1 then implies

$$(-1)m = -m.$$

□