**Proposition 1.14:** For all $m \in \mathbb{Z}$, $m \cdot 0 = 0 = 0 \cdot m$.

*Proof*. Let $m \in \mathbb{Z}$. Since multiplicative commutativity implies $m \cdot 0 = 0 \cdot m$, it is enough to show that $m \cdot 0 = 0$. Notice that

$$\begin{aligned} m \cdot 0 &= m \cdot (0 + 0) && \text{(Axiom 1.2)} \\ & m \cdot 0 + m \cdot 0 && \text{(distributivity).} \end{aligned}$$

But then Axiom 1.2 implies

$$m \cdot 0 + 0 = m \cdot 0 + m \cdot 0.$$

So, by Proposition 1.9,

$$0 = m \cdot 0.$$

□

**Proposition 1.17(i):** The integer 0 is divisible by every integer.

*Proof*. Let $n \in \mathbb{Z}$. To show that $n \mid 0$, we must find an integer $j$ such that

$$0 = j \cdot n. \tag{1}$$

Proposition 1.14 implies

$$0 = 0 \cdot n.$$

Hence equation (1) holds with $j = 0$. □