Proposition 1.7: If m is an integer, then $0 + m = m$ and $1 \cdot m = m$.

Proof. Let $m \in \mathbb{Z}$. Then by additive commutivity

$$0 + m = m + 0.$$

But $m + 0 = m$ by Axiom 1.2. So $0 + m = m$.

On the other hand, by multiplicative commutivity,

$$1 \cdot m = m \cdot 1.$$

Axiom 1.3 implies $m \cdot 1 = m$, so $1 \cdot m = m$. □

Proposition 1.8: If m is an integer, then $(-m) + m = 0$.

Proof. Let $m \in \mathbb{Z}$. Then additive commutativity implies

$$(-m) + m = m + (-m).$$

Since Axiom 1.4 implies $m + (-m) = 0$, it follows that $(-m) + m = 0$. □

Proposition 1.11(iii): Let m, n and p be integers. Then $m + (n + p) = (p + m) + n$.

Proof. Suppose m, n, and p are integers. Then

$$\begin{aligned} m + (n + p) &= m + (p + n) && \text{by additive commutativity} \\ &= (m + p) + n && \text{by additive associativity} \\ &= (p + m) + n && \text{by additive commutativity.} \end{aligned}$$

□