If A and B are sets, we say they have the *same cardinality* if there exists a bijection $f : A \to B$, in which case we write $|A| = |B|$. We say a set A is *countable* if there exists a bijection $f : \mathbb{N} \to A$.

Proposition HW14.1: Cardinality defines an equivalence relation.

Proof. □

Proposition HW14.2: The set $(0, 1)$ has the same cardinality as $(-1, 1)$.

Proof. □

Proposition HW14.3: The set $(0, 1)$ has the same cardinality as $(0, \infty)$.

Proof. □

Proposition HW14.4: The set $(-1, 1)$ has the same cardinality as $\mathbb{R}$.

Proof. □

Proposition HW14.5: The set $(0, 1)$ has the same cardinality as $(0, 1]$.

Proof. □

Proposition HW14.6: $\{a + \sqrt{2}b : a, b \in \mathbb{Q}\}$ is countable.

Proof. □