Proposition HW13.1: Suppose $f : \mathbb{Z} \to \mathbb{R}$ and that for all $n, m \in \mathbb{Z}$, $f(n + m) = f(n) + f(m)$ and $f(n \cdot m) = f(n) \cdot f(m)$. Then either $f(n) = 0_{\mathbb{R}}$ for all $n \in \mathbb{Z}$ or $f = e$.

Proof. Your proof here. Hint: either $f(1_{\mathbb{Z}}) = 0_{\mathbb{R}}$ or not. □

In chapter 9 we constructed a function e from $\mathbb{Z}$ to $\mathbb{R}$ that obeyed nice rules with respect to arithmetic. The point of proposition 9.G is that there was only one good way to do this.

Proposition 11.17: Suppose $x, y \in \mathbb{R}$ and $x < y$. Then there exists an irrational number z such that $x < z < y$.

Proof. Your proof goes here. □

Proposition 10.B: Suppose $x \in \mathbb{R}$ and $M \geq 0$. Then $|x| \leq M$ if and only if $-M \leq x \leq M$.

Proof. Your proof goes here. □

Proposition HW 13.2: $\mathbb{N} \subseteq \mathbb{R}$ is not bounded above.

Proof. Your proof goes here. Hint: If $\mathbb{N}$ were bounded above, it would have a supremum x. I bet there's a natural number larger than $x - \frac{1}{2}$ then... □