

This looks long, but most of the proofs are very short!

Proposition 8.A: The number $0 \in \mathbb{R}$ does not have a multiplicative inverse.

Proof. □

Proposition 8.B: If $c, x \in \mathbb{R}$ and $cx = 1$, then $x \neq 0$ and $c = x^{-1}$.

Proof. □

Proposition 8.C: If $x, y \in \mathbb{R}$ and $x \neq 0$ and $y \neq 0$, then $xy \neq 0$ and $(xy)^{-1} = x^{-1}y^{-1}$.

Proof. □

Proposition 8.D: If $x \in \mathbb{R}$ and $x \neq 0$, then $x^{-1} \neq 0$ and $(x^{-1})^{-1} = x$.

Proof. □

Proposition 8.E: If $x \in \mathbb{R}$ and $x > 0$, then $x^{-1} > 0$.

Proof. □

Corollary 8.F: If $x \in \mathbb{R}$ and $x \neq 0$, if $x^{-1} > 0$ then $x > 0$.

Proof. □

Proposition 8.40:

(ii) Let $x, y \in \mathbb{R}$ such that $0 < x < y$. Then $0 < 1/y < 1/x$.

Proof. Be sure to take advantage of Propositions 2A-2H as well as the work you've just done to make for a very short proof. □

Proposition 8.43: Let $x, y \in \mathbb{R}$ such that $x < y$. Then there exists $z \in \mathbb{R}$ such that $x < z < y$.

Proof. □

Proposition 8.45: If x_1 and x_2 are least upper bounds for $A \subseteq \mathbb{R}$, then $x_1 = x_2$.

Proof. □

Proposition 8.45: $\sup((-\infty, 0)) = 0$

Proof. □

Lemma 9.A: Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$.

1. If $g \circ f$ is injective, then f is injective.
2. If $g \circ f$ is surjective, then g is surjective.
3. If $g \circ f$ is bijective, then f is injective and g is surjective.

Proof. □

Proposition 9.7: (ii) If $f : A \rightarrow B$ is surjective, and $G : B \rightarrow C$ is surjective, then $g \circ f : A \rightarrow C$ is surjective.

Proof. □

Proposition 9.11: If $f : A \rightarrow B$ has an inverse function, the inverse function is unique.

Proof. □

Proposition 11.3: If $x, y, z \in \mathbb{R}$ with $y \neq 0$ and $z \neq 0$, then

$$\frac{xz}{yz} = \frac{x}{y}.$$

Proof. Your proof goes here. □