Axiom 1.1-1.5: Axioms regarding the integers: arithmetic, additive and multiplicative identities, additive inverses, and multiplicative cancellation.

Proposition 1.6: If m, n, and p are integers, then $(m + n) \cdot p = m \cdot p + n \cdot p$.

Proposition 1.7: If m is an integer, then $0 + m = m$ and $1 \cdot m = m$.

Proposition 1.8: If m is an integer, then $(-m) + m = 0$.

Proposition 1.9: Let m, n, and p be integers. If $m + n = n + p$ then $n = p$.

Proposition 1.10: Let $m, x_1, x_2 \in \mathbb{Z}$. If $m + x_1 = 0$ and $m + x_2 = 0$ then $x1 = x2$.

Proposition 1.11: If m, n, p, and q are integers then

(i) $(m + n)(p + q) = (mp + np) + (mq + nq)$

(ii) $(m + n) + (p + q) = m + (n + (p + q)) = ((m + n) + p) + q$

(iii) $m + (n + p) = (p + m) + n$

(iv) $m(np) = p(mn)$

(v) $m(n + (p + q)) = (mn + mp) + mq$

(vi) $(m(n + p))q = (mn)q + m(pq)$

Proposition 1.12: Let $x \in \mathbb{Z}$. If x has the property that for each integer m, $m + x = m$, then $x = 0$.

Proposition 1.13: Let $x \in \mathbb{Z}$. If x has the property that there exists an integer m such that $m + x = m$, then $x = 0$.

Proposition 1.14: For all $m \in \mathbb{Z}$, $m \cdot 0 = 0 \cdot m = 0$.

Proposition 1.16: If m and n are even integers, then so are $m + n$ and $m \cdot n$.

Proposition 1.17:

(i) 0 is divisible by every integer.

(ii) If m is an integer not equal to 0, then m is not divisible by 0.

Proposition 1.18: Let $x \in \mathbb{Z}$. If x has the property that for each integer m, $m \cdot x = m$, then $x = 1$.

Proposition 1.19: Let $x \in \mathbb{Z}$. If x has the property that there exists a non-zero integer m such that $m \cdot x = m$, then $x = 1$.

Proposition 1.20: For all $m, n \in \mathbb{Z}$, $(-m) \cdot (-n) = m \cdot n$.

Corollary 1.21: $(-1) \cdot (-1) = 1$.

Proposition 1.22:

(i) For all $m \in \mathbb{Z}$, $-(-m) = m$.

(ii) $-0 = 0$.

Proposition 1.23: Given $m, n \in \mathbb{Z}$, there exists one and only on $x \in \mathbb{Z}$ such that $m + x = n$

Proposition 1.24: Let $x \in \mathbb{Z}$. If $x \cdot x = 0$ then $x = 0$ or $x = 1$.

Proposition 1.25: For all $m, n \in \mathbb{Z}$.

(i) $-(m + n) = (-m) + (-n)$

(ii) $-m = (-1) \cdot m$

(iii) $(-m)n = m(-n) = -(mn)$

Proposition 1.26: Let $m, n \in \mathbb{Z}$. If $mn = 0$ then $m = 0$ or $n = 0$.

Proposition 1.27: For all $m, n, p, q \in \mathbb{Z}$:

(i) $(m - n) + (p - q) = (m + p) - (n + q)$

(ii) $(m - n) - (p - q) = (m + q) - (n + p)$

(iii) $(m - n)(p - q) = (mp + nq) - (mq + np)$

(iv) $m - n = p - q$ if and only if $m + q = n + p$

(v) $(m - n)p = mp - np$