Here are some more things you should know for the midterm, which will cover Chapter 3, and Sections 2.7 and 4.1.

What is the rank of a matrix? How do you compute it?

Text: 3.5 number 25.

What is the definition of linear independence? Know **both** of the definitions in the text, but especially the formulation in terms of linear combinations (i.e. the book's second definition).

What's a basis?

What does it mean that $\mathbf{v}_1$ through $\mathbf{v}_n$ span W? Note that it does not mean "everything in W is a linear combination of the $\mathbf{v}_i$'s". One thing is missing.

What's a basis of W?

How do you find a basis for the column space of A?

How do you find a basis for the null space of A?

How do you find a basis for the cokernel of A?

How do you find a basis for the row space of A?

If $A \sim U \sim R$, how are the four fundamental subspaces of A related to the echelon matrix U and the reduced echelon matrix R?

Given 4 vectors in $\mathbb{R}^5$, how can you determine if they are linearly independent or not? Determine if the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 1 \\ 5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ 4 \\ 2 \\ 1 \\ 9 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -2 \\ 2 \\ -1 \\ 4 \\ 6 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 4 \\ 1 \\ 5 \\ 6 \\ 2 \end{bmatrix},$$

are linearly independent. Hint: convert the problem to one of finding a null space. Then let Matlab/Octave do the hard work.

Find a basis in $\mathbb{R}^4$ for the space spanned by $(14, 63, 7, 28)$, $(18, -4, 14, 31)$, $(10, 11, 7, 18)$. Convert this problem to the problem of finding the column space of some matrix. Now let Matlab/Octave do the work.

Important Each of the subparts of 3.5 number 16 can be phrased as finding a basis for a null or column space of some matrix. Find the four matrices, and indicate which space (null or column) applies.

Let W be the intersection of the hyperplanes in $\mathbb{R}^5$ determined by

$$2x + y - z + 9u - 5v = 0$$

and

$$z + u - 4v = 0.$$

Find a basis for W by converting the problem to finding some null or column space. What dimension is W?

Suppose A is an invertible $n \times n$ matrix. Explain carefully why $\mathrm{Col}(A) = \mathbb{R}^n$. Explain carefully why the columns are linearly independent. Use these two facts to conclude that the columns of A are a basis for $\mathbb{R}^n$.

Suppose A is an $m \times n$ matrix. If you do elimination on A, how can you spot if $\mathrm{Col}(A) = \mathbb{R}^m$ or not?

Suppose $\mathbf{v} = (2, 1, 4, 3)$ and $\mathbf{w} = (2, 1, 4, -3)$. Let

$$A = \mathbf{v}\mathbf{w}^T.$$

Find a basis for each of the four fundamental subspaces of A. Hint: the column space and row spaces should be easy!

Suppose $A\mathbf{x} = \mathbf{b}$ and $\mathbf{y}$ is in the cokernel of A. Show that $\mathbf{y} \cdot \mathbf{b} = 0$.

Worked example 3.6 A. Know especially how to compute a basis for the cokernel.

What are the dimensions of the 4 fundamental spaces?

Section 3.6, problem 14.

Section 3.6, problem 21.

Section 3.6, problem 23.