Exercise 1: Suppose we have two points $\mathbf{p}_1 = (x_1, y_1)$ and $\mathbf{p}_2 = (x_2, y_2)$ and we would like to find the equation of a line $y = mx + b$ going through those points. Substitute these points into the equation of the line to obtain two equations for the unknowns m and b.

Exercise 2: Suppose $\mathbf{p}_1 = (0.2, 0.7)$ and $\mathbf{p}_2 = (0.8, -0.4)$. What are the equations for m and b?

Exercise 3: Write these equations in matrix form:

$$A\begin{bmatrix} m \\ b \end{bmatrix} = \mathbf{v}.$$

Explicitly write down what A and $\mathbf{v}$ are.

Exercise 4: Solve these equations as follows. First, enter the matrix A into Octave. Then enter the right-hand side $\mathbf{v}$. Form the augmented matrix A_{aug} by entering `Aaug=[A,v]`. Solve the system using `M=rref(Aaug)`. Then extract m and b from their locations in M.

Exercise 5: Verify your solution is correct by making the a plot that contains your line (using the values of m and b that you computed), and contains the two points plotted as circles.

Exercise 6: Now find the equation of a parabola $y = ax^2 + bx + c$ passing through the points $(-1, 1.5)$, $(3, 32.2)$, $(5, -42.6)$. You must

a) Record the system of equations to solve.

b) Convert the system into a matrix system.

c) Solve the system using Octave (record the command you used and the solution).

d) Generate a plot that contains the parabola and the three points.

Exercise 7: Explain, in terms of intersecting planes, why if you have only two data points, these determine infinitely many parabolas.

Exercise 8: Explain, in terms of intersecting planes, why if you have four data points, you expect there will be no parabola containing all four points.

Exercise 9: If you have 7 data points (x_n, y_n), with all of the x_n's different, these uniquely determine a polynomial of some order. What is the order?

Exercise 10: Find the equation of a polynomial of the kind discussed in the previous step that passes through the points $(1, 4)$, $(2, -1)$, $(4, 7)$, $(5, -3)$, $(8, 1)$, $(9, -10)$ $(11, 3)$. You will find the following Ocatave command to be helpful: If $\mathbf{c} = (c_1, c_2, \ldots, c_n)$ has been entered into MATLAB as the column vector `c`, then the command `vander(c)` will return the matrix

$$\begin{bmatrix} c_1^{n-1} & c_1^{n-2} & \ldots & c_1 & 1 \\ c_2^{n-1} & c_2^{n-2} & \ldots & c_2 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_n^{n-1} & c_n^{n-2} & \ldots & c_n & 1 \end{bmatrix}.$$

This is called a *Vandermonde matrix*.) Record the formula for your polynomial, together with a graph of it and the data points.