The matrix problems we have solved by hand in this class have all been cooked to keep the computations as simple as possible. This is done so that you can focus on the big picture for the most part and avoid the inevitable computational errors that will happen with messier problems. A downside to having saved you from unpleasant computation, however, is that you might have a skewed perspective on the difficulty of actually doing these computations. Now you might think, "Well, I never have to do these computations: I can use a computer". But even with computers, we want to keep the level of work required to a minimum so that we can solve large problems. Efficiency counts, even for computers. The point of the following exercises is to do some more realistic (but still very easy problems) by hand to get a better sense of how much work the computations require, and what a savings an orthonormal basis is (if you have one). Please do all the work by hand (with a calculator). You should verify your work as often as you want with Matlab to verify that you aren't making mistakes. But the exercises will be most meaningful if you do the computations honestly by hand.

1. The vectors

$$\mathbf{v}_1 = \begin{bmatrix} -13.43652 \\ 7.24364 \\ -1.41259 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 11.83359 \\ 8.32167 \\ 2.79195 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} -6.25518 \\ 0.62153 \\ -9.65021 \end{bmatrix}$$

form a basis for $\mathbb{R}^3$. Express $\mathbf{b} = (2, 4, 2)$ in terms of this basis. That is, find a way to write $\mathbf{b} = x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3$ for some scalars x_1, x_2, and x_3.

2. To see the computational ease of working with an orthonormal basis, consider

$$\mathbf{q}_1 = \begin{bmatrix} -0.876491 \\ 0.472517 \\ -0.092146 \end{bmatrix} \quad \mathbf{q}_2 = \begin{bmatrix} -0.454419 \\ -0.875237 \\ -0.165720 \end{bmatrix} \quad \mathbf{q}_3 = \begin{bmatrix} -0.158955 \\ -0.103379 \\ 0.981858 \end{bmatrix}$$

This is an orthonormal basis for $\mathbb{R}^3$. Express $\mathbf{b}$ in terms of this basis.

3. Let V be the subspace spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$. Decompose $\mathbf{b}$ into $\mathbf{b}_1 + \mathbf{b}_2$ where $\mathbf{b}_1 \in V$ and $\mathbf{b}_2 \in V^\perp$. Work by hand using the basis of $\mathbf{v}_i$'s.

4. The basis $\mathbf{q}_1$, $\mathbf{q}_2$, and $\mathbf{q}_3$ was obtained from $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ by the Gram-Schmidt procedure. So $\mathbf{q}_1$ and $\mathbf{q}_2$ are also a basis for V. Use this basis to compute $\mathbf{b}_1$ and $\mathbf{b}_2$. (Hint: this problem is either easy, or ridiculously easy).