The final exam will cover all the material from the two midterms and, additionally, all of Chapter 4, all of Chapter 5, and Sections 6.1 and 6.2. Material from the labs is examinable. Everything from the homework and the quizzes is fair game. Please study homework or quiz problems that you did not get right the first time. Please go back and look over the study guides for the midterms. Here are more study ideas.

Suppose A is an $m \times n$ matrix with rows $\mathbf{w}_1^T$ through $\mathbf{w}_m^T$. Suppose $\mathbf{x}$ is a linear combination of $\mathbf{w}_1$ through $\mathbf{w}_m$ and that $\mathbf{n}$ is in the null space of A. Show that $\mathbf{x} \cdot \mathbf{n} = 0$. You should show this directly without citing the FTLA part II.

In problem 4.1.6, find a basis for the cokernel of A. Then demonstrate that the equation is not solvable using your basis.

Suppose A is symmetric (A^T=A), $A\mathbf{x} = 2\mathbf{x}$, and $A\mathbf{y} = 5\mathbf{y}$. Show that $\mathbf{x}^T\mathbf{y} = 0$. (Eigenvectors of symmetric matrices are perpendicular!)

Exercise 4.1.11

Let W be the span of $(1, 2)$. Let $\mathbf{b} = (1, 1)$. Write $\mathbf{b}$ as a sum of an element of W and an element of $W^\perp$.

If W is the span of vectors $\mathbf{v}_1$ to $\mathbf{v}_n$ in $\mathbb{R}^m$, and $\mathbf{b}$ is some point in $\mathbb{R}^m$, how do you find the closest point in W to $\mathbf{b}$? You should be able to do this both by computing the projection matrix, and without computing the projection matrix explicitly. For practice, try 4.2.16.

How is this procedure simplified if $\mathbf{v}_1$ to $\mathbf{v}_n$ are orthonormal?

How is this procedure simplified if $\mathbf{v}_1$ to $\mathbf{v}_n$ are only orthogonal?

Exercise 4.2.17

Exercise 4.2.29. Hint: If A is a square matrix, it is invertible if and only if the only solution of $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$.

Section 4.3 Problems 16 and 18

Section 4.4.5 First find a basis for the plane W. Then use Gram-Schmidt to find the orthonormal basis. For extra fun, project $(1, 1, 1)$ into W using your orthonormal basis. Then find a basis for $W^\perp$.

You must know all of the properties of the determinant from Section 5.1. The most important are $|I| = 1$, determinants change sign when two rows are interchanged, and multilinearity. We proved the rest of the properties from these three. Can you use these to show that a matrix with a zero row must have determinant zero?

Prove that $|A^{-1}| = 1/|A|$.

Suppose P is a projection, i.e. $P^2 = P$. Find the possible values for $|P|$? Suppose $A^T = A^{-1}$. What are the possible values for $|A|$?

Problem 5.1.13

Problem 5.1.29

Problem 5.1.34

How do you compute the sign of a permutation?

What is the definition of minor M_{ij}? Of cofactor C_{ij}?

I liked all those problems using a recursion formula to compute the determinants of highly symmetric banded matrices!

Go look at the solution to Quiz 12. Then use this technique to compute the determinant of the matrices in problem 5.2.4.

How do you compute eigenvectors and eigenvalues of a 2×2 or 3×3 matrix?

How is the graph of the discrete cosine transform of a function related to the graph of the original time series?