Math F200X Midterm 1 Spring 2010

Name: _____

Id: _____

Rules:

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

All derivatives must be computed directly from the definition using limits.

No calculators, books, notes, or other aids are permitted.

Turn off anything that might go beep during the exam.

If you need extra space, you can use the back sides of the pages. Please make it obvious when you have done so.

If you need to compute a derivative, you may use our rules for taking derivatives **unless** the problem asks you specifically to compute the derivative using the (limit) definition of the derivatives.

Good luck!

Problem	Possible	Score
1	15	
2	5	
3	20	
4	20	
5	20	
6	6	
7	14	
Total	100	

1. (15 points)

Compute the following limits.

a $\lim_{x \to 2} \cos(x) \frac{x^2 - 4}{x - 2}$

b $\lim_{x \to 2} \frac{x^2 - 4}{|x - 2|}$

c $\lim_{x \to 3^-} \frac{x + 2}{x - 3}$

2. (5 points)

What is the inverse function of the function $f(x) = e^{x^3 + 2}$?

3. (20 points)

A Fairbanks resident is filling up his water tank at the local water station. The volume of water V in the tank at time t is

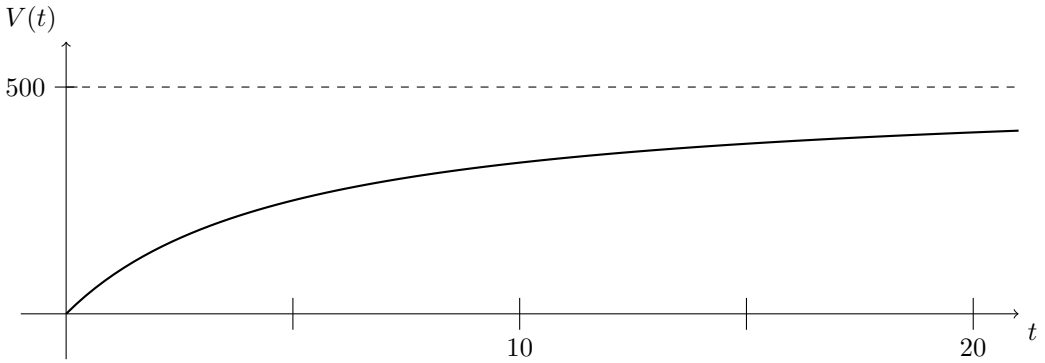
$$V(t) = 500\frac{t}{5+t}$$

where volume is measured in gallons and time is measured in minutes.

a How much water is in the tank at time $t = 5$? What are the units of your answer?

b What is the average rate of change of the amount of water in the tank over the time interval from $t = 5$ to $t = 10$? What are the units of your answer?

c In the graph of $V(t)$ below, draw a secant line such that the slope of the secant line is equal to the average rate of change you found in part b.



d How fast is the tank being filled at the instant $t = 5$? What are the units of your answer?

4. (20 points)

a Let $f(x) = \sqrt{x}$. Use the limit definition of the derivative to compute $f'(3)$.

b What is the equation for the tangent line to the graph of $y = f(x)$ at $x = 3$?

5. (20 points)

Compute the following derivatives.

a $\dfrac{d}{dx}\,(x^4+9)\cos(x)$

b $\dfrac{d}{dx}\,\dfrac{e^x+x^2}{\sqrt{x}+1}$

c $\dfrac{d}{dx}\,\sin(x)\left(1+\dfrac{1}{x}\right)e^x$

d $\dfrac{d}{dx}\,\dfrac{1}{x\sqrt{x}}$

6. (6 points)

The limits below are the derivatives of some function $f(x)$ at some point $x = a$. Determine what $f(x)$ and a are in each case. **Do not evaluate the limit!**

a $\lim_{h \to 0} \dfrac{\sin(5+h) - \sin(5)}{h}$

b $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

7. (14 points)

The axis on the left shows the graph of a function $f(x)$. In the axis on the right, sketch the graph of $f'(x)$.