Recall that we have two equivalent formulations for the derivative of a function.

$$f'(a) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a} \tag{1}$$

and

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}. \tag{2}$$

The first is a limit of slopes of secant lines of the graph of $f(x)$ between $x = a$ and $x = b$ as $b \to a$. Then second is a limit of slopes of secant lines of the graph of $f(x)$ between $x = a$ and $x = a + h$ as $h \to 0$. In both cases, one endpoint is at $x = a$, and the other end point is brought closer and closer to a.

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1. Consider the function $f(x) = \sqrt{x}$.

a) Compute the slopes of the secant lines of the graph of $f(x)$ between $x = 2$ and $x = b$ for $b = 2.1$, $b = 1.99$, and $b = 1.999$.

b) Compute the slopes of the secant lines of the graph of $f(x)$ between $x = 2$ and $x = 2 + h$ for $h = .1$, $h = -0.01$, and $h = -0.001$.

c) Your numbers above estimate the value of $f'(x)$ at $x = c$ for some number c. What is the value of c?

d) Using only your computations above, estimate for $f'(c)$.

e) Compute $f'(c)$ exactly using derivative rules. (Here c is the number you found in part c. above)

2. Each of the limits in Section 2.3, problem numbers 81, 83, 84, 85, 87, and 89 can be written as $f'(c)$ for some some function $f(x)$ at some point $x = c$. For each of these 5 limits, identify the function $f(x)$ and the point c. For example, if one of the limits was

$$\lim_{h \to 0} \frac{\sin(h)}{h}$$

I would notice that $\sin(h) = \sin(0 + h) - 0 = \sin(0 + h) - \sin(0)$. So this limit is in fact:

$$\lim_{h \to 0} \frac{\sin(0 + h) - \sin(0)}{h}$$

This matches the format of equation (2), and hence this is the derivative of $f(x) = \sin(x)$ at $x = 0$. For full credit, you must rewrite each limit in the form of equation (1) or equation(2), just like I did here.

3. Compute the following derivatives; on Monday's lecture we'll introduce the tool you need.

a) $\dfrac{d}{dx} \dfrac{1}{1 + 3x}$

b) $\dfrac{d}{dx}\left[\, 2(3x-\pi)^{14} - 6(2-x)^{8}\right]$

c) $\dfrac{d}{dx}\,\cos\left(\dfrac{2\pi}{30}(x-10)\right)$