1. Ice is growing over a pond. The thickness of the ice is given by $D(t) = \sqrt{t}$ where D is measured in centimeters and $t \geq 0$ is measured in days.

a) What is the average rate of change of the ice thickness over the time interval $t \in [1, 1+h]$? What are the units of your answer?

b) Write the value of the instantaneous rate of change of the ice thickness at time $t = 1$ as a limit involving your answer to part a).

c) Compute the value of the limit to exactly compute the instantaneous rate of change at time $t = 1$.

d) Repeat the previous steps for time $t = 0$ rather than time $t = 1$.

e) Sketch the curve $D(t) = \sqrt{t}$ and explain why your answer to part d) is reasonable.

2. Consider the function $f(x) = |x \sin(x)|$.

a) Sketch the graph of $f(x)$ for $-2\pi \leq x \leq 2\pi$.

b) Why is $f(x) \leq |x|$?

c) Why is $f(x) \geq 0$?

d) On the same graph as part a), add the graphs of functions $g(x) = |x|$ and $h(x) = 0$. Add these two graphs in a different color.

e) Compute $\lim_{x\to 0} g(x)$ and $\lim_{x\to 0} h(x)$.

f) What does the Squeeze Theorem now imply?

3. Let $f(x) = |x|\cos(x)$. Follow a similar recipe to problem 3 to use the Squeeze Theorem to compute $\lim_{x\to 0} f(x)$. You must figure out functions $g(x)$ and $h(x)$ for yourself and explain why $g(x) \leq f(x) \leq h(x)$. You should also graph all three functions for $-2\pi \leq x \leq 2\pi$.

4. Suppose

$$f(x) = \begin{cases} 3x - 1 & x < 1 \\ (x-a)^2 & x \geq 1 \end{cases}$$

where a is some number that you don't know. Compute $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$. For what choices of a is $f(x)$ continuous?