Math F200X Final Exam Spring 2008

Name: _____

Id: _____

Rules:

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

No calculators, books, notes, or other aids are permitted.

Turn off anything that might go beep during the exam.

If you need extra space, you can use the back sides of the pages. Please make it obvious when you have done so.

Good luck!

Problem	Possible	Score
1	16	
2	16	
3	30	
4	15	
5	15	
6	10	
7	15	
8	15	
9	10	
10	12	
Total	154	

1. (16 points)

Compute the following derivatives.

a. $\dfrac{d}{dx}\ \sin\left(\sqrt{1+x^2}\right)$

b. $\dfrac{d}{dx}\left(\dfrac{x^2+1}{x^4+2}\right)^{50}$

c. $\dfrac{d}{d\theta}\ 3\tan^2(\theta) - \sec^2(\theta)$

d. $\dfrac{d}{dx}\ x^{\sin(x)}$

2. (16 points)

Compute the following integrals.

a. $\int_0^2 x^3 - \sqrt{x}\, dx$

b. $\int_0^1 3^x\, dx$

c. $\int \sqrt{x}\cos(2 + x^{3/2})\, dx$

d. $\int_e^{e^2} \frac{1}{x\ln(x)}\, dx$

3. (30 points)

Let $f(x) = xe^{-x^2}$.

a. Does the function have any symmetry? If so, what?

b. On what intervals is $f(x)$ increasing/decreasing?

c. What are the critical points of $f(x)$?

d. Classify each critical point as a local maximum or local minimum.

e. On what intervals is $f(x)$ concave up/concave down?

f. Find any points of inflection of $f(x)$.

g. Compute $\lim_{x\to\infty} f(x)$.

h. Use the information you have determined above to sketch the graph of $f(x)$.

4. (15 points)

a. Find the linear approximation of the function $f(x) = \tan\left(\frac{\pi}{4}e^x\right)$ at $x = 0$.

b. Use your answer from part **a.** to estimate $f(0.1)$.

c. Consider the curve defined implicitly by $x^3 - y^3 = 2xy$. Find the slope of the tangent line to this curve at the point $(-1, 1)$.

5. (15 points)

A pizza delivery company has determined that if the price of a pizza is x dollars, then it can sell

$$N(x) = 1000 - 240\ln(x/12)$$

pizzas in Fairbanks on a Friday night.

a. If the price of pizzas is \$12, how many pizzas will be sold on a Friday night and how much money will be made?

b. Let $R(x)$ be the amount of revenue generated (i.e. money made) if the price of pizzas is x dollars. Express $R(x)$ as a function of x.

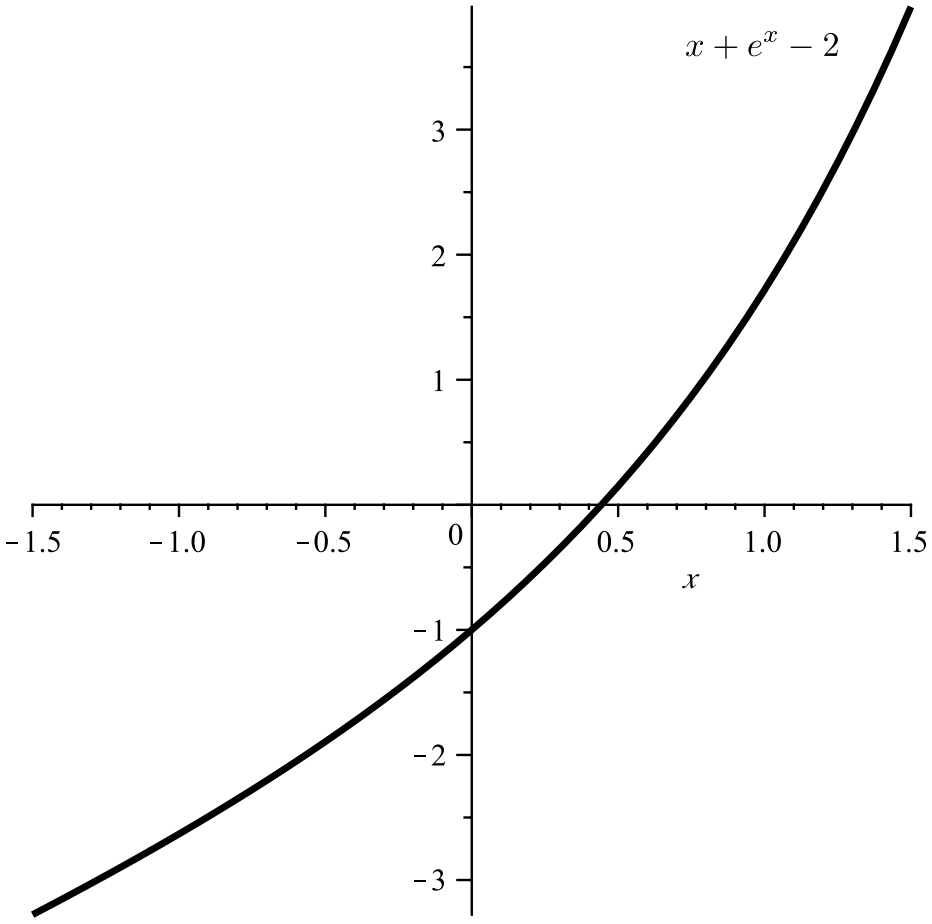
c. Find the maximum amount of revenue the pizza company can make on a Friday night. Be sure to demonstrate that you have found a **global** maximum.

6. (10 points)

a. Let $f(x) = x + e^x - 2$. Apply **one** iteration of Newton's method with an initial guess $x_0 = 1$ to find a subsequent approximation x_1.

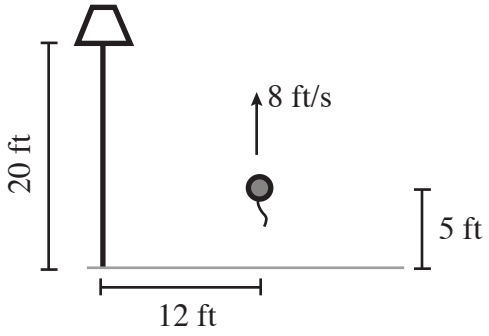
b. Draw a diagram, using the graph of $f(x)$ below, to clearly explain how the value x_1 was obtained from x_0.

c. In the diagram below, indicate the approximate position of x_{100}.



7. (15 points)

A helium balloon is rising vertically at a rate of 8ft/sec. The balloon is located 12 feet away from a 20 foot lamp post. How fast is the shadow of the balloon moving away from the lamp post when the balloon is 5 feet high?



8. (15 points)

The population of aardvarks in a certain region is given by a function $P(t)$ where time is measured in years and the population is measured in hundreds of aardvarks. It is known that for $t \geq 0$, the rate of change of the population is given by

$$P'(t) = te^{-t^2}.$$

a. What is the change in the number of aardvarks from time $t = 1$ to time $t = 2$?

b. Suppose that $P(0) = 3$. Determine $P(t)$.

c. What is the eventual population of the aardvarks?

9. (10 points)

Compute the following limits.

a. $\lim_{x \to 3} \dfrac{\arctan(3) - \arctan(x)}{3 - x}$

b. $\lim_{x \to \infty} \dfrac{x^3 + x\cos(\sqrt{x}) + 1}{x - 2x^3}$

c. $\lim_{x \to 0^+} x^2 \ln(x)$

d. $\lim_{x \to 1^+} \dfrac{x - 2}{(x-1)^2}$

e. $\lim_{n \to \infty} \sum_{i=1}^{n} e^{i/n} \dfrac{1}{n}$

10. (12 points)

Let $\mathcal{R}$ be the region in the plane bounded by the curves $y = 1$, $x = 2$, and $y = e^{-x}$.

a. Sketch the region $\mathcal{R}$.

b. Write down a definite integral that expresses the volume of the region obtained by rotating $\mathcal{R}$ about the line $y = 2$. **Do not evaluate the integral.**

c. Write down a definite integral that expresses the volume of the region obtained by rotating $\mathcal{R}$ about the line $x = 0$. **Do not evaluate the integral.**