1. Suppose $f \in B[a,b]$ and $c \in [a,b]$. Show that $f \in \mathrm{Riem}[a,b]$ if and only if $f \in \mathrm{Riem}[a,c]$ and $f \in \mathrm{Riem}[c,b]$, in which case

$$\int_a^b f = \int_a^c f + \int_c^b f.$$

2. Show that integration on $\mathrm{Riem}[a,b]$ is continuous by showing that the map

$$f \mapsto \int_a^b f$$

is a bounded linear map.

3. Determine if $\chi_\Delta \in \mathrm{Riem}[0,1]$, where Δ is the Cantor set.

4. Suppose $l : \mathcal{P}(\mathbb{R}) \to [0,\infty]$ is monotone. Show that l is countably additive if and only if l is finitely additive and countably subadditive. Feel free to quote results from class for parts of this problem.

5. 16.4

6. 16.12

7. 16.16

8. 16.24

9. 16.25

10. 16.28