1. Carothers 11.52

2. Carothers 11.58

3. Let $K(x,t)$ be a continuous function on $[0,1]\times[0,1]$. For $f \in C[0,1]$ define a function $T(f)$ on $[0,1]$ by $(Tf)(x) = \int_0^1 K(x,y)f(y)\,dy$.

a) Show that $Tf \in C[0,1]$ if $f \in C[0,1]$.

b) Suppose (f_n) is a bounded sequence in $C[0,1]$. Show that (Tf_n) has a (uniformly) convergent subsequence.

4. Suppose $f \in \mathrm{Riem}[a,b]$ and $\alpha \in \mathbb{R}$. Show that $\alpha f \in \mathrm{Riem}[a,b]$ and

$$\int_a^b \alpha f = \alpha \int_a^b f$$

5. Show that the uniform limit of Riemann integrable functions is Riemann integrable. Conclude that $\mathrm{Riem}[a,b]$ is a closed subset of $B[a,b]$.

6. Find (with proof) an element of $\mathrm{Riem}[a,b]$ that is not a uniform limit of step functions.