1. Let A be a continuous operator on the Hilbert space X. Show $(A^*)^* = A$.

2. D&M 4.4b, c (Note that we effectively already proved part a early on in the semester.)

3. D & M 4.23

4. Suppose A is a continuous operator on the Hilbert space X. Show

$$\mathrm{Image}(A)^\perp = \mathrm{Ker}(A^*)$$

and

$$\mathrm{Ker}(A)^\perp = \overline{\mathrm{Image}(A^*)}$$

5. Let I be a bounded open interval and let $f \in L^1(I)$. Suppose $\int f\psi = 0$ for all $\psi \in \mathcal{D}(I)$.

a) Conclude that $\int_J f = 0$ for all closed intervals $J \subseteq I$. (Hint: Homework 8, problem 6).

b) Conclude that $\int_E f = 0$ for all measurable sets $E \subseteq I$. (Hint: Approximate E with finitely many closed intervals and take advantage of Carothers 18.17, which you have already proved.)

c) Conclude that $f = 0$ almost everywhere on I.