1. Let X be a Hilbert space and $S \subseteq X$. Show that if for each $f \in X^*$ the set $\{f(x) : x \in X\}$ is bounded, then S is bounded.

2. Let X be a vector space. If W is a subspace of X we can put an equivalence relation on X by $x \sim y$ if $x - y \in W$ (or alternatively if $x = y + w$ for some $w \in W$). We write equivalence classes as $x + W$ rather than $[x]$. The set of equivalence classes is denoted X/W. We can put a vector space structure on X/W by $(x + W) + (y + W) = (x + y) + W$ and $\lambda(x + W) = (\lambda x) + W$. You are invited to prove to yourself (but not to me) that these operations are well-defined and that X/W becomes a vector space with these operations.

Now suppose further that X is a normed space and W is a closed subspace of X.

a) Show that X/W is a normed space with $||x + W||_{X/W} = \inf_{y \in x+W} ||y||_X$.

b) Show that if X is a Banach space, then so is X/W. (Hint: Use Banach's characterization of complete spaces.)

3. Suppose $T : X \to Y$ is a continuous surjective linear map between Banach spaces. Show that $X/\operatorname{Ker} T$ is isomorphic as a Banach space to Y. That is, there is a continuous linear bijection $S : X/\operatorname{Ker} T \to Y$ that has a continuous inverse.

4. Suppose X is a closed subspace of $L_2[0,2]$ and that for every $f \in L_2[0,1]$ there is an $F \in L_2[0,2]$ such that $F|_{[0,1]} = f$. Show that there is a $c > 0$ such that we can pick F such that $||F||_{L_2[0,2]} \le c||f||_{L_2[0,1]}$.