Many problems on the current assignment use the Baire category theorem, which is not in your text. We presented two versions in class today. Here's a third formulation that might be helpful.

Proposition 1. *Let X be a complete metric space. If $\{F_i\}$ is a sequence of closed sets in X and if $X = \cup_i F_i$ then at least one F_i has non-empty interior.*

This is little more than a recapitulation of the statement that a complete metric space is not meager.

1. (Solution by David Maxwell)
Suppose that X is a real normed vector space that satisfies the parallelogram law. We will show that

$$\langle x, y \rangle = \frac{1}{4}\left[||x+y||^2 - ||x-y||^2 \right]$$

is an inner product on X that generates its norm.

Let $a, b, y \in X$.

a) Use two applications of the parallelogram law to show

$$\langle a+b, y \rangle = 2\langle a, y/2 \rangle + 2\langle b, y/2 \rangle .$$

b) Conclude that $\langle a+b, y \rangle = \langle a, y \rangle + \langle b, y \rangle$.

c) Show that $\langle ka, y \rangle = k\langle a, y \rangle$ for all $k \in \mathbb{N}$, and then for all $k \in \mathbb{Q}$.

d) Justify the conclusion that $\langle \alpha a, y \rangle = \alpha \langle a, y \rangle$ for all $\alpha \in \mathbb{R}$.

2. (Solution by TJ Barry)
Show that a complete metric space without any isolated points is uncountable.

3. (Solution by Vikenty Mikeev)
Let X be a Hilbert space and suppose $P : X \to X$, $P^2 = P$, and $||P|| = 1$. Show that P is the orthogonal projection on to $P(X)$.

4. (Solution by David Maxwell)
Show that the plane is not a countable union of lines.

5. (Solution by Lyman Gilispie)
Show that a normed vector space with an countably infinite algebraic basis is not complete.

6. (Solution by Will Mitchell)
Let X be a normed space and $S \subseteq X$. Show that if for each $f \in X^*$ the set $\{f(x) : x \in X\}$ is bounded, then S is bounded.