

1. (Solution by TJ Barry)

Suppose p, q and r belong to $[1, \infty]$ and that

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1,$$

where $1/\infty = 0$. Show that if $x, y, z \in \mathbb{C}^n$, then

$$\sum_{k=1}^n |x_k y_k z_k| \leq \|x\|_p \|y\|_q \|z\|_r.$$

2. (Solution by Lyman Gilispie)

Let Z be the set of sequences that are eventually zero. Show that if $p \neq q$ then the ℓ_p and ℓ_q norms on Z are not equivalent.

3. (Solution by Will Mitchell)

Suppose W is an open subspace of the normed vector space X . Show that $W = X$. Then exhibit an example of a subspace of a normed vector space that is not closed.

4. (Solution by Vikenty Mikheev)

D&M 1.30

5. (Solution by David Maxwell)

D&M 1.26

6. (Solution by David Maxwell)

D&M 1.47