1. (Solution by TJ Barry)
Suppose $K \in L^2([0,1] \times [0,1])$. Define $T : L^2([0,1] \to L^2([0,1]))$ by

$$(Tf)x = \int_0^1 K(x,y)f(y)\,dy.$$

Show that T is compact. Feel free to use the results of Examples 4.2.4 and 4.8.4 (which we effectively proved in class).

2. Let X be a Hilbert space. We say a linear map T is bounded below if there exists a constant c such that $||Tx|| \geq c||x||$ for all $x \in X$. Show that if $T : X \to X$ is linear and continuous, it is invertible if and only if it is bounded below and onto a dense subspace.

3. (Solution by Will Mitchell)
Let $\{\lambda_k\}$ be a bounded sequence. Define $T : \ell_2 \to \ell_2$ by

$$T(x) = (\lambda_1 x_1, \lambda_2 x_2, \ldots).$$

It is easy to see that T is continuous; don't show this. Instead, determine $\sigma_p(T)$ and $\sigma(T)$.

4. (Solution by David Maxwell)
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