As in the first lab, please work in groups of three students on this lab. It might be wise to ensure that at least one member of your group has a little bit of programming experience.

The lab is due Wednesday, October 19. Note that the week the lab is due, there will also be two regular assignments homework. Therefore you need to get started on the lab **now**.

Answer each of the questions below. If a question asks you to describe or explain something, you need to use full sentences. The grading will take into account the clarity of your answers as well as their grammatical correctness.

Your project must be typed up and then printed out. Using Word is fine. Math and physics students might be interested in trying out LaTeX. Computer generated figures can be attached to the end of your document so long as they are clearly labeled and referenced.

1. **By hand**, use the improved Euler method to approximate the solution of

$$y' = 3(9 + 10\cos(2\pi t) - y), \quad y(0) = 12 \tag{1}$$

on the time interval $0 \le t \le 3$ with **just one** time step.

Hand In:

1. Your work showing the values of t and y at the timesteps.

2. Make a function `ImprovedEuler.m` that works just like your `Euler.m` function but implements the improved Euler method. Verify that your code works by checking that it agrees with your work in problem 1.

Hand In:

1. A copy of your `ImprovedEuler.m` code.
2. A copy of the Octave output showing your test comparison with problem 1.

3. Graph the exact solution of equation (1) and your approximation using 100 time steps all on the plot. It might be handy to go back and look at your solution to Homework 6, problem 6.

Hand In:

1. A copy of the plot. It should clearly shows which graph is the approximation and which graph is the exact solution.

4. Suppose $y(t) = Ct^m$ where C and m are constants. What do you get if you plot $\log(y)$ versus $\log(t)$? Hint: let $Y = \log(y)$ and $T = \log(t)$, and write Y as a function of T.

Hand In:

1. A clear and lucid explanation of what the resulting graph is and how its features are related to the values of C and m.

5. In Octave, you can plot $\log(y)$ versus $\log(x)$ using the `loglog` function: `loglog(x,y)`. Generate log-log graphs of the following functions all on one graph: $y = x^{-1/2}$, $y = x$, and $y = x^2$. Then generate log-log graphs of the following functions all on one graph: $y = x^{-1/2}$, $y = 2x^{-1/2}$, $y = 3x^{-1/2}$.

Hand In:

1. The two plots.
2. A brief, but precise, description of how the three curves on each of your plots are related to each other.

6. In this problem we compare the error made by the Euler and ImprovedEuler methods, again in computing the solution to

$$y' = 3(9 + 10\cos(2\pi t) - y), \quad y(0) = 12. \tag{2}$$

Use both methods to approximate $y(3)$ using $N = 100$, $N = 500$, $N = 1000$, $N = 5000$ and $N = 10000$ timesteps. How many digits of accuracy do you obtain? (You might find the command `format long` helpful in showing more digits). Plot the log of the (absolute value of the) error versus $\log(h)$ for both methods on the same chart. You should see the graphs of two lines. What are the slopes of those lines? What does this say about how the error for the two methods is related to the step-size?

Hand In:

1. A handwritten or printout copy of your approximations of $y(3)$ using both methods, with all five values of N.
2. The log-log plot of error versus h.
3. A printout of the Matlab/Octave code you used to make the plot.
4. A lucid paragraph explaining what the slopes of the lines are, and what they mean in terms of how the error of the two methods is related to the step size h.

7. On the course web page you will find code for `MysterySolver.m` that takes the same inputs and yields the same output variables as `Euler.m` and `ImprovedEuler.m`. How is the error for this ODE solver related to the step size h? Perform a similar analysis as you did in the previous problem.

Hand In:

1. A description of how error and step size are related for this method, along with any supporting plots and computations.