Proposition 6.5: Assume we are given an equivalence relation on a set A. For all $a_1, a_2 \in A$, either $[a_1] = [a_2]$ or $[a_1] \cap [a_2] = \emptyset$.

Proof. Your proof goes here. □

Proposition 6.6 (Partial)**:** Let A be a set and let Π be a partition of A. We define $a \sim b$ if there exists $P \in \Pi$ such that $a \in P$ and $b \in P$. Then $\sim$ is an equivalence relation.

Proof. Your proof goes here. □

Project 6.7: For each of the following relations defined on $\mathbb{Z}$, determine whether it is an equivalence relation. If it is, determine its equivalence classes.

1. $x \sim y$ if $x < y$.
2. $x \sim y$ if $x \leq y$.
3. $x \sim y$ if $|x| = |y|$.
4. $x \sim y$ if $x \neq y$.
5. $x \sim y$ if $xy > 0$.
6. $x \sim y$ if $x \mid y$ or $y \mid x$.

Proposition 6.17: Let $m \in \mathbb{Z}$. Then m is even if and only if m^2 is even.

Proof. Your proof goes here. □

Proposition 6.25: If $a \equiv a' \pmod{n}$ and $b \equiv b' \pmod{n}$ then

$$a + b \equiv a' + b' \pmod{n}$$

and

$$ab \equiv a'b' \pmod{n}.$$

Proof. Your proof goes here. □

Project 6.27: Study the set n such that $\mathbb{Z}_n$ satisfies the cancellation property (Axiom 1.5). You should form a conjecture, and then prove it.

Proposition 6.28: Every integer greater than or equal to 2 can be factored in to primes.

Proof. Your proof goes here. □