

Proposition 4.8: For all $k \in \mathbb{N}$, $4^k > k$.

Proof. Your proof goes here. □

Proposition 4.13: For $x \neq 1$ and $k \in \mathbb{Z}_{\geq 0}$, $\sum_{j=0}^k x^j = \frac{1 - x^{k+1}}{1 - x}$.

Hint: Show that $(1 - x) \sum_{j=0}^k x^j = 1 - x^{k+1}$.

Proof. Your proof goes here. □

Proposition 4.15(i): Let $m \in \mathbb{Z}$ and $(x_j)_{j=1}^{\infty}$ be a sequence in \mathbb{Z} . If then for all $k \in \mathbb{N}$

$$\sum_{j=1}^k mx_j = m \sum_{j=1}^k x_j.$$

Proof. Your proof goes here. □

Proposition 4.15(iii): Let $(x_j)_{j=1}^{\infty}$ be a sequence in \mathbb{Z} . If $x_j = n \in \mathbb{Z}$ for all $j \in \mathbb{N}$ then for all $k \in \mathbb{N}$

$$\sum_{j=1}^k x_j = kn.$$

Proof. Your proof goes here. □

Proposition 4.16(ii): Let $(x_j)_{j=m}^{\infty}$ and $(y_j)_{j=m}^{\infty}$ be sequences in \mathbb{Z} . For all $a, b \in \mathbb{Z}$ such that $m \leq a \leq b$,

$$\sum_{j=a}^b (x_j + y_j) = \sum_{j=a}^b x_j + \sum_{j=a}^b y_j.$$

Proof. Your proof goes here. □

Proposition 4.18: Let $(x_j)_{j=1}^{\infty}$ and $(y_j)_{j=1}^{\infty}$ be sequences in \mathbb{Z} such that $x_j \leq y_j$ for all $j \in \mathbb{N}$. Then for all $k \in \mathbb{N}$,

$$\sum_{j=1}^k x_j \leq \sum_{j=1}^k y_j.$$

Proof. Your proof goes here. □

Proposition 4.30: For all $k, m \in \mathbb{N}$, where $m \geq 2$,

$$f_{m+k} = f_{m-1}f_k + f_m f_{k+1}.$$

Proof. Your proof goes here.

□