Proposition 4.6(iii): Let $b \in \mathbb{Z}$ and $m, k \geq 0$. Then $(b^m)^k = b^{mk}$.

Proof. □

Proposition 4.11: For all $k \in \mathbb{N}$,

$$2 \sum_{j=1}^{k} j = k(k+1).$$

Proof. □

Proposition 4.A: Suppose a and b are integers such that $a \neq 0$ and $a \mid b$. Then there exists a unique integer j such that $b = aj$.

Proof. □

There is nothing more to prove on this homework. The discussion below explains how to rewrite the result of Proposition 4.11 more naturally using Proposition 4.A.

Definition: Suppose that a and b are integers such that $a \neq 0$ and $a \mid b$. We define

$$\frac{b}{a} = j$$

where j is the unique integer such that $b = aj$

If a, b, and c are integers (with $a \neq 0$) and if we write

$$\frac{b}{a} = c$$

we mean a divides b and $b = ca$. To show that $\frac{b}{a} = c$ you simply show that $b = ac$. With this definition in mind, Proposition 4.11 can be rephrased

$$\sum_{j=1}^{k} j = \frac{k(k+1)}{2}.$$