Proposition 9.18: For all $m, n \in \mathbb{Z}$,

$$e(m \cdot n) = e(m) \cdot e(n).$$

Proof. Your proof goes here. Attempt maximal laziness. □

Proposition 9.19: If $n, m \in \mathbb{Z}$ then

$$n < m \quad \text{if and only if} \quad e(n) < e(m).$$

Proof. For full credit, you must be maximally lazy. Use Lemma 9.E proved in class, and use a string of if and only ifs. □

Proposition 11.3: If $x, y, z \in \mathbb{R}$ with $y \neq 0$ and $z \neq 0$, then

$$\frac{xz}{yz} = \frac{x}{y}.$$

Proof. Your proof goes here. □

Lemma 11.A: Suppose $a, b \in \mathbb{Z}$ and $b \neq 0$. Let $g = \gcd(a, b)$, so $g \in \mathbb{N}$, $g \mid a$ and $g \mid b$. Then

$$\gcd\left(\frac{a}{g}, \frac{b}{g}\right) = 1.$$

Proof. Recall that if $c \mid d$, and $c \neq 0$, then $\frac{c}{d}$ is the unique integer j such that $c = jd$.

Feel free to use Proposition 6.30; I'll give a proof in class. □

Proposition 11.4: Let $x \in \mathbb{Q}$. Then there exist integers a and b such that $b > 0$, $\gcd(a, b) = 1$, and $x = a/b$.

Proof. The proof is not long if you take advantage of Proposition 11.3 and Lemma 11.A. Ignore the hint in the book; it will likely lead to a hand-wavy proof. □

The following proposition might take you some work. Please start on it now. It will not be due until the next week, however.

Proposition 9.G: Suppose $f : \mathbb{Z} \to \mathbb{R}$ and that for all $n, m \in \mathbb{Z}$, $f(n + m) = f(n) + f(m)$ and $f(n \cdot m) = f(n) \cdot f(m)$. Then either $f(n) = 0_{\mathbb{R}}$ for all $n \in \mathbb{Z}$ or $f = e$.

Proof. Your proof here. Hint: either $f(1_{\mathbb{Z}}) = 0_{\mathbb{R}}$ or not. □

In chapter 9 we constructed a function e from $\mathbb{Z}$ to $\mathbb{R}$ that obeyed nice rules with respect to arithmetic. The point of proposition 9.G is that there was only one good way to do this.