

The final exam will be held on May 7, 2012 and will be comprehensive.

The exam will be closed book. I will provide for you a list of any relevant propositions from the text and the homework, except:

- I will **not** provide for you the statement of any Axiom.
- I will **not** provide for you the statement of any Theorem.
- I will **not** provide for you the statements of any Proposition that is very closely related to a Theorem or an Axiom.
- I will **not** provide for you the statements of any Proposition that is not fundamental or is irrelevant to the exam.

Reasonable tasks for the exam include (but are not limited to):

- Prove a brand new result that is not in the text but can be proved from our propositions.
- Prove a result from the text.
- State major definitions, theorems, and axioms.
- State the “first line of the proof.”
- Given the statement of a proposition, set up the proof using the contrapositive or converse or weak or strong induction (without actually finishing the proof).
- Given a flawed proof, find the error and fix it.

Since the second midterm, the main topics we have covered have been: the real numbers, the embedding of the integers in the real numbers, the completeness of the real numbers, the existence of square roots, and cardinality. You should know the key distinctions between the integers and the real numbers. What axioms do they have in common. What’s true about the integers that’s not true about \mathbb{R} ? What about the other way around.

Regarding cardinality, you should be aware of the following results:

Subsets of finite sets are finite.

Subsets of countable sets are countable.

A set A is countable if and only if there is a surjection $f : \mathbb{N} \rightarrow A$. Moreover, if B is countable and $f : B \rightarrow A$ is a surjection, then A is countable.

Countable unions of countable sets are countable.

Finite products of countable sets are countable.

\mathbb{R} and $\{0, 1\}^\omega$ are uncountable.

Mathematicians live and die by definitions. You have to know them. Here are some definitions you need to know (regardless of whether they show up on the midterm or not). I have only listed things here that were not listed in previous study guides.

- The axioms for \mathbb{R} .
- Upper and lower bounds for subsets of \mathbb{R} .
- The supremum of a subset of \mathbb{R} .
- The rational numbers.
- Two sets having the same cardinality.
- A finite set. An infinite set. A countably infinite set. A countable set. An uncountable set.