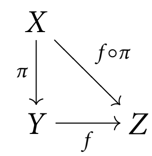
1. Suppose X is a second countable space. Let $\{U_\alpha\}_{\alpha\in I}$ be a collection of open sets in X such that $\cup_\alpha U_\alpha = X$. (We call $\{U_\alpha\}_{\alpha\in I}$ an *open cover* of X.) Prove that there exists a countable sub-collection $\{U_{\alpha_i}\}$ such that $\cup_i U_{\alpha_i} = X$. (We say that every open cover of a second countable space has a countable subcover).

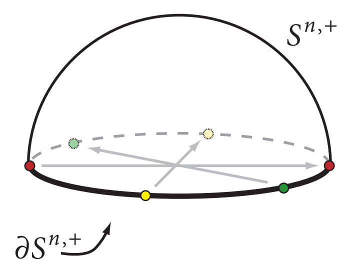
2. Suppose π is a quotient map from a second countable space X to a locally Euclidean space Y. Prove that Y is second countable. *Hint:* Problem **1** might be handy.

3. (The Characteristic Property of the Quotient Topology is characteristic.) Suppose $\pi : X \to Y$ is a surjective function. Suppose that for any topological space Z that a function $f : Y \to Z$ is continuous if and only if $f \circ \pi$ is. Show that Y has the quotient topology induced by π.



4. Show that the upper half sphere $S^{n,+}$ with antipodal points on $\partial S^{n,+}$ identified is homeomorphic to $\mathbb{RP}^n$.

5. Show that $\mathbb{RP}^n$ is an n-manifold. You are free to use any facts we have proved in class or on the homeworks in your proof. There are several approaches to this problem; try to find the shortest proof you can.



Gluing a half sphere.

6. Munkres 22.4: No rigor, please!

7. Munkres 19.7