1. Show that if X is a first countable space, and if convergent sequences in X have unique limits, then X is Hausdorff.

2. Let X, Y, and Z be topological spaces. Show that $(X \times Y) \times Z$ is homeomorphic to $X \times Y \times Z$.

3. Show that the product of an n-manifold N and an m-manifold M is an $n + m$ manifold.

4. Munkres 17.13

5. Munkres 17.9

6. Show that a topological space is a 0-manifold if and only if it is at most countable and has the discrete topology.

7. A set $A \subseteq X$ is said to be *dense* in X if $\bar{A} = X$. A topological space is *second countable* if it admits a countable basis.

a) Suppose X is second countable. Prove that it has a countable dense subset.

b) Prove that a metric space is second countable if and only if it has a countable dense subset.

8. Let $\pi : X \to Y$ be a surjective continuous map. Show that π is a quotient map if and only if it takes saturated closed sets to saturated closed sets.

9. Let $\pi : X \to Y$ be a quotient map. Show that if A is a saturated open set or a saturated closed set, then $\pi|_A : A \to \pi(A)$ is a quotient map.

10. 22.3

11. Let X be the line with two zeros. That is, X is the quotient space of $\{0,1\} \times \mathbb{R}$ given by the equivalence relation $(0,x) \sim (1,x)$ if $x \neq 0$. Show that X is locally Euclidean and second countable, but not Hausdorff.