For this assignment, feel free to use Maple to help you out with any problem you like. Procedures you've written in the past should be helpful.

1. Consider the cone $\mathbf{x}(u,v) = (u\cos(v), u\sin(v), u)$.

a) Compute its first fundamental form (i.e. the matrix of I with respect to the basis $\mathbf{x}_u$, $\mathbf{x}_v$).

b) Consider the curve given by $(u(t), v(t)) = (e^{\lambda t}, t)$ where λ is a constant. Find the length of this curve for $0 \leq t \leq \pi$.

c) Use Maple to plot this surface and this curve on the surface with $\lambda = 1$.

2. Consider the chart $\mathbf{x}(u,v) = (u, v, u^2 + v^2)$. Compute its second fundamental form (i.e. the matrix of II with respect to the basis $\mathbf{x}_u$, $\mathbf{x}_v$.)

3. Suppose that $\mathbf{x}$ is a surface chart with $II(\mathbf{v}, \mathbf{w}) = 0$ for all tangent vectors $\mathbf{v}$ and $\mathbf{w}$ at each point of the surface. Prove that the surface patch is contained in a plane.

4. Assume that a surface M satisfies $|k_1| \leq 1$ and $|k_2| \leq 1$ everywhere. Is it true that the curvature κ of a curve in M satisfies $\kappa \leq 1$ everywhere?

5. Write a procedure in Maple that will compute the principal curvatures and principal vectors of a surface chart $\mathbf{x}$. Use your procedure to compute the principal curvatures of the following charts.

a) The helicoid: $\mathbf{x}(u,v) = (v\cos u, v\sin u, bu)$

b) The catenoid: $\mathbf{x}(u,v) = (\cosh u\cos v, \sin v, u\cos v)$

6. Oprea 2.4.4