1. Do exercise Oprea 2.2.15 **without** using Maple in any way.

2. Let V be a two-dimensional inner-product space. Let $\{v_1, v_2\}$ be a basis for V, though perhaps not an orthonormal basis. Let $x \in V$, so we can write $x = a_1 v_1 + a_2 v_2$ for some coefficients a_i. In general it is hard to determine what these coefficients are. For example, if the inner-product space is $\mathbb{R}^2$ with the dot-product, $v_1 = (1,3)$, $v_2 = (-4,7)$, and $x = (5,2)$, what are a_1 and a_2? In this problem we determine a procedure for doing this by using the inner product.

   Define $g_{ij} = \langle v_i, v_j \rangle$. Show that (a_1, a_2) solves the matrix equation

$$\begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

   where $b_i = \langle x, v_i \rangle$.

   Use this procedure to determine the coefficients a_1 and a_2 when $v_1 = (1,3)$, $v_2 = (-4,7)$ and $x = (5,2)$. You may use Maple to help you with the computation, if you want. (Check out the LinearSolve command).

3. The tangent space of a surface is an inner product space. We define $\langle \mathbf{v}, \mathbf{w} \rangle = \mathbf{v} \cdot \mathbf{w}$ where for the dot product we think of the vectors as belonging to $\mathbb{R}^3$. Let $\mathbf{x}$ be a chart into a surface and suppose $\mathbf{v} = a\mathbf{x}_u + b\mathbf{x}_v$. Show that $\langle \mathbf{v}, \mathbf{v} \rangle = Ea^2 + 2Fab + Gb^2$ where

$$E = \langle \mathbf{x}_u, \mathbf{x}_u \rangle \qquad F = \langle \mathbf{x}_u, \mathbf{x}_v \rangle \qquad G = \langle \mathbf{x}_v, \mathbf{x}_v \rangle .$$

   (Notice: in the notation of problem 2, $E = g_{11}$, $F = g_{12} = g_{21}$, $G = g_{22}$. The notation E, F, and G is older and is traditional for surfaces.)

4. Compute E, F, and G for the chart $\mathbf{x}(u,v) = (\cos(u)\sin(v), \sin(u)\sin(v), \cos(v))$. You may use Maple to assist you in your computation. Explain why your results for F and G make intuitive sense. Extra Credit: Write a procedure in Maple that given a chart and parameter names returns the square matrix $\begin{pmatrix} E & F \\ F & G \end{pmatrix}$.

5. Let $\mathbf{x}$ be a chart with domain D into the surface M. Let $\tilde{\alpha} : [0, T] \to D$ be a curve in D, so $\tilde{\alpha}(t) = (u(t), v(t))$ for some functions u and v. Let $\alpha(t) = \mathbf{x}(\alpha(t))$, so α is a curve in M. Show that

$$L(\alpha) = \int_0^T E(\tilde{\alpha}(t))(u'(t))^2 + 2 * F(\tilde{\alpha}(t))u'(t)v'(t) + G(\tilde{\alpha}(t))(v'(t))^2 \, dt.$$

   This shows us that E, F, and G can be used to compute the lengths of curves in local coordinates.

   Let $\mathbf{x}$ be the chart in problem **??**. Let $\tilde{\alpha}(t) = (t, \pi/4)$ where $-\pi < t < \pi$. Use the formula developed above and the values of E, F, and G computed in problem **??** to compute the length of α. Explain why the result of this computation makes intuitive sense.

6. Write a Maple procedure **UnitNormal** that takes an expression for a chart $\mathbf{x}$ and the name of two coordinate variables (e.g. u and v) and returns a simplified expression for the unit normal in terms of u and v. Verify your procedure works by computing the unit normal of the helicoid $\mathbf{x}(u,v) = (v\cos u, v\sin u, bv)$. (You have already computed this normal on a previous homework.)

7. Write a Maple procedure **Shape** that takes an expression for a chart $\mathbf{x}$ and the name of two coordinate variables (e.g. u and v) and returns the matrix of the shape operator with respect to the basis $\mathbf{x}_u$ and $\mathbf{x}_v$. You will find problem 2 to be essential in solving this problem.

8. Use the program you wrote in the previous problem to do Exercise 2.2.16 in Oprea.