All parts of this homework to be completed in Maple should be done in a single worksheet. You can submit either the worksheet by email or a printout of it with your homework.

1. Consider the lima con $\alpha(t) = ((1+2\cos t)\cos t, (1+2\cos t)\sin t)$, which has period 2π. Plot this curve using Maple. Compute its curvature and show that α has exactly two vertices. You may use Maple to assist in your computations, but you must also explain your conclusion in words (either on paper or in a Maple text area).

2. Let α be a simple closed unit-speed positively oriented plane curve. Let r be a positive constant scalar. The curve

$$\beta(t) = \alpha(t) + rN_s(t)$$

is called a parallel curve to α. One can show that if r is sufficiently small, then β is also a simple positively oriented curve (but not unit speed). (You need not prove this!).

a) Show that $L(\beta) = L(\alpha) + 2\pi r$.

b) Show that $\text{Area}(\beta) = \text{Area}(\alpha) + rL(\alpha) + \pi r^2$

c) Verify these formulas for $\alpha(t) = (\cos(t), \sin(t))$.

Hint: You will need both the Theorem of Turning Tangents, and our formula for the area inside a curve

$$\begin{aligned} 2\text{Area} &= \int_C x\,dy \;\; - y\,dx \\ &= \int_0^L (-y(t), x(t)) \cdot (x'(t), y'(t))\,dt \\ &= \int_0^L (J\alpha(t)) \cdot \alpha'(t)\,dt \end{aligned}$$

where J is the rotation matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

3. Oprea 2.1.11 (There is a mistake in this question – part of your job is to find the mistake and fix it.)

4. Oprea 2.1.13

5. Oprea 2.1.15

6. Oprea 2.1.20. Also, make a plot in Maple demonstrating that the saddle is a ruled surface.