All parts of this homework to be completed in Maple should be done in a single worksheet. You can submit either the worksheet by email or a printout of it with your homework.

**1.** Let $k(s)$ be a smooth function on $\mathbb{R}$. Let

$$\theta(s) = \int_0^s k(u)\,du$$

and

$$\alpha(s) = \left( \int_0^s \cos(\theta(u))\,du, \int_0^s \sin(\theta(u))\,du \right).$$

Show that $\alpha$ is a smooth unit speed curve with signed curvature $\kappa_s(s) = k(s)$.

**2.** Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a translation (so $T(x) = x + v$ for some constant vector $v$). Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be a rotation. Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection $S(x, y) = (-x, y)$. If $\alpha$ is a smooth plane curve, show that the signed curvatures of $T \circ \alpha$ and $R \circ \alpha$ are the same as those of $\alpha$, but the signed curvature of $S \circ \alpha$ is the negative of the signed curvature of $\alpha$.

**3.** Use Maple to plot the trace of a plane curve with signed curvature $\kappa_s(s) = 2\sin(s)$. Explain why $\alpha(2\pi)$ lies on the $x$-axis.

**4.** Let $\alpha$ be a unit speed plane curve. Its center of curvature is

$$\epsilon(s) = \alpha(s) + \frac{1}{\kappa_s(s)} N(s).$$

a) Show that the circle centered at $\epsilon(s)$ is tangent to $\alpha$ at $\alpha(s)$ and has the same curvature as $\alpha$ at that point. You should use facts you know about the curvature of a circle.

b) The curve $\epsilon(s)$ is called the evolute of $\alpha$. Show that the unit tangent to $\epsilon$ is $N(s)$ and the signed unit normal to $\epsilon$ is $-T$.

c) Let $v$ be the arclength parameter of $\epsilon$. Show that

$$\frac{dv}{ds} = \left| \frac{\kappa_s'(s)}{\kappa^2} \right|$$

d) Compute the signed curvature of $\epsilon(s)$.

**5.** Exercise 1.3.23.

**6.** (This problem to be done entirely with Maple.) Viviani's curve is defined by

$$\alpha(t) = (\cos(t)^2 - 1/2, \sin(t)\cos(t), \sin(t)).$$

a) Show that $\alpha$ lies on the sphere of radius 1 centered at $(-1/2, 0, 0)$ and on the cylinder $x^2 + y^2 = 1$.

b) Make a plot in Maple to demonstrate that $\alpha$ lies on this sphere. The commands **plots**[**spacecurve**], **plottools**[**sphere**] and **plottools**[**display**] might come in handy. Also note that if you end a line in Maple with a colon rather than a semicolon, the output will be suppressed, which is handy for things like the output of **plottools**[**sphere**].

c) Compute the curvature and torsion of Viviani's curve.

d) Verify that the curvature and torsion of Viviani's curve satisfy the formula

$$R^2 = (1/\kappa)^2 + ((1/\kappa)'(1/\tau))^2$$

from the previous problem.