1. Find an isometry from a part of the xy-plane to the cone

$$\mathbf{x}(u,v) = (u\cos v, u\sin v, u)$$

where $u > 0$ and $0 < v < 2\pi$.

2. We have a chart into the helicoid given by $\mathbf{x}(u,v) = (u\cos v, u\sin v, v)$. We have a chart into the catenoid given by $\mathbf{y}(u,v) = (\cosh u\cos v, \cosh u\sin v, u)$. In these charts we restrict $0 < v < 2\pi$.

Show that the map taking $\mathbf{y}(u,v)$ to $\mathbf{x}(\sinh u, v)$ is an isometry.

3. Define charts $\mathbf{x}$ and $\mathbf{y}$ as in the previous problem For each t, define

$$\mathbf{x}^t = \cos t\mathbf{y}(u, v+t)\sin t\mathbf{x}(\sinh u, v+t-\pi/2)$$

Show that $\mathbf{x}^0 = \mathbf{y}$, $\mathbf{x}^{\pi/2} = \mathbf{y}$ and that for each t, show that the map taking $\mathbf{x}(u,v)$ to $\mathbf{x}^t(u,v)$ is a (local) isometry.

For extra credit, use Maple to generate an animation showing this deformation (by isometries) from the catenoid to the helicoid.

4. (Stereographic coordinates)
Let M be the sphere $x^2+y^2+z^2=1$, and let $N=(0,0,1)$ be the north pole. Given a point p in the plane, we let q be the point on S on the line segment from N to p. For example, if $p=(0,0,0)$, then $q=(0,0,-1)$.

a) Let $p=(u,v,0)$. Find a formula for q in terms of u and v.

b) Show that the map taking p to q is a conformal map.

5. Mercator's parameterization of the sphere is given by $\mathbf{x}(u,v) = (\operatorname{sech} u\cos v, \operatorname{sech} u\sin v, \tanh u)$. A chart into the cylinder is given by $\mathbf{y}(u,v) = (\cos v, \sin v, u)$. Show that the map taking $\mathbf{y}(u,v)$ to $\mathbf{x}(u,v)$ is a conformal map.