

1. Write a Maple procedure that takes as an argument a chart $\mathbf{x}$, two parameter names (e.g. u, v), an initial point (in (u, v) coordinates) and an initial velocity (u', v') and returns a procedure that gives a curve (in your choice of (u, v) or (x, y, z) coordinates) that represents the corresponding geodesic. You will want to use Maple's `dsolve` command with the `numeric` option here.
2. Use Maple to generate plots of geodesics on the one-sheeted hyperboloid $x^2 - z^2 = 1$ with angular velocities $\Omega = 0.5, 0.8, 1$ (both kinds), 1.2, and 2.
3. Use the Clairaut relation to give a qualitative description of all the geodesics on the torus. Be sure to discuss all cases of values of the angular momentum.
4. A chart into the pseudosphere is given by

$$\mathbf{x}(u, v) = \left[\frac{1}{u} \cos(v), \frac{1}{u} \sin(v), \sqrt{1 - \frac{1}{u^2}} - \operatorname{arccosh}(u) \right]$$

where $u > 1$ and $-\pi < v < \pi$. (The constraint on v can be relaxed, but the constraint on u is required for smoothness).

- a) Show (by hand or otherwise) that for this chart $E = F = 1/w^2$ and $G = 0$.
- b) Show that every geodesic in local coordinates is either contained in a line $v = v_0$ or satisfies

$$(v - v_0)^2 + u^2 = \frac{1}{\Omega^2}$$

where Ω is the angular momentum of the geodesic.

- c) Draw what geodesics look like in local coordinates, and use Maple to generate plots of some corresponding geodesics on the pseudosphere.

5. Oprea 5.2.14