1. Let (x_n) be a sequence. Prove that $\lim x_n = x$ if and only if for any open set G containing x, there is an N such that if $n \geq N$, $x_n \in G$.

2. Let G be an open set and let $x_1, \ldots, x_n$ be finitely many points in G. Prove that $G \setminus \{x_1, \ldots, x_n\}$ is open. *Hint:* : Use Theorem 3.2.3.

3. Abbott 3.2.2 (Don't worry about being too rigorous for this one)

4. For each of the following sets, determine if it is open, closed, or neither. Be rigorous here, but don't do much work. Be sure to take advantage of Theorem 3.2.3, Theorem 3.2.13 and Exercises 5 and 6 below. You may freely used the fact that an open interval (a, b) is open, and a closed interval $[a, b]$ is closed.

a) $\mathbb{Q}$

b) $\mathbb{Z}$

c) $\{x \in \mathbb{R} : x > \pi\}$

d) $(0, 1] = \{x : 0 < x \leq 1\}$

e) $\{1 + 1/4 + 1/9 + \cdots + 1/n^2 : n \in \mathbb{N}\}$

5. Let $A \subseteq \mathbb{R}$. Prove that A is closed if and only if every convergent sequence (a_n) with each $a_n \in A$ converges to a limit in A. You may not cite Theorem 3.2.8 (unless you choose to prove it.)

6. Abbott 3.2.13 **(Hand this one in to David.)**

7. Abbott 4.2.1 a, b

8. Abbott 4.2.3