First, some notation that you might find useful.

Let P and Q be points in the plane, let $\mathcal{L}_{PQ}$ be the equidistant line between them. Given a line $\mathcal{L}$, let $R_{\mathcal{L}}$ be the reflection about $\mathcal{L}$. Finally, let R_{PQ} be the reflection about $\mathcal{L}_{PQ}$.

In the following we will consider an isometry F and three non-colinear points A, B, C. Let $\hat{A} = F(A)$, $\hat{B} = F(B)$, and $\hat{C} = F(C)$.

1. What is $R_{PQ}(P)$?

2. Find a reflection R_1 that takes A to $\hat{A}$. Consider both the case $A = \hat{A}$ and the case $A \neq \hat{A}$.

3. Explain why $\hat{B}$ and $R_1(B)$ lie on a circle centered at $\hat{A}$.

4. Find a reflection R_2 taking $R_1(B)$ to $\hat{B}$. There are two cases.

5. What is $R_2(\hat{A})$? Why?

6. Explain why $\hat{C}$ and $R_2(R_1(C))$ are on a circle centered at $\hat{B}$, and are also on a circle centered at $\hat{A}$.

7. Find a reflection R_3 taking $R_2(R_1(C))$ to $\hat{C}$. There are two cases.

8. What is $R_3(\hat{A})$? What is $R_3(\hat{B})$?

9. Let $G = R_3 \circ R_2 \circ R_1$. How are G and F related? Why?

10. Find reflections R_1 and R_2 such that $R_2 \circ R_1$ is a translation of distance 2 along the x-axis.