1. Let $\mathcal{L}_1$ be the line $x = 1$. Let $\mathcal{L}_2$ be the line $x = k$, where $k \neq 0$. If you project $\mathcal{L}_1$ onto $\mathcal{L}_2$ through the origin, the point $(1, z)$ on $\mathcal{L}_1$ will be taken to a point $(k, f_k(z)$ on $\mathcal{L}_2$. Determine the function $f_k(z)$.

2. In the previous projection, where does the point at $\infty$ on $\mathcal{L}_1$ get taken to?

3. Let $\mathcal{L}_1$ be the line $x = 1$. Let $\mathcal{L}_2$ be the line $y = 1$. If you project $\mathcal{L}_1$ onto $\mathcal{L}_2$ through the origin, the point $(1, z)$ on $\mathcal{L}_1$ will be taken to a point $(g(z), 1)$ on $\mathcal{L}_2$. Determine the function $g(z)$.

4. In the previous projection, where does the point at $\infty$ on $\mathcal{L}_1$ get taken to? What gets taken to the point at $\infty$ on $\mathcal{L}_2$? Explain you answer both in terms of algebra and in terms of geometry.

5. Let $\mathcal{L}_1$ be the line $x = 1$. Let $\mathcal{L}_2$ be the line $x = 2$. Imagine projecting $\mathcal{L}_1$ onto $\mathcal{L}_2$ from a point at infinity in the direction $(1, d)$. The point $(1, z)$ on $\mathcal{L}_1$ will be taken to a point $(2, h_d(z))$ on $\mathcal{L}_2$. Determine the function $h_d(z)$.

6. In the previous projection, where does the point at $\infty$ on $\mathcal{L}_1$ get taken to?

7. What is the most general function you can make composing functions of the form $f_k$ and $h_d$ (as often as you like)?

8. What do the functions of your previous answer do to the point at $\infty$?

9. What is the most general function you can make composing functions of the form $f_k$, $h_d$, and $g$ (as often as you like)? It can be shown that every projection from a line to another can be written in this form.

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10. Consider a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. It defines a map from $\mathbb{R}^2$ to itself by matrix multiplication. Explain how you can also think about this as defining a map from $\mathbb{RP}^1$ to itself.

11. Find two different matrices that represent the same map from $\mathbb{RP}^1$ to itself. Can you conjecture when $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$ represent the same map?

12. What is the general form of a matrix that sends the point with homogeneous coordinates $(0, 1)$ to itself?

13. What is the general form of a matrix that leaves both the points with homogeneous coordinates $(0, 1)$ and $(1, 0)$ fixed?

14. What is the general form of a matrix that leaves the points with homogeneous coordinates $(0, 1)$, $(1, 0)$, and $(1, 1)$ fixed?

**15.** One way to understand the action of a map on $\mathbb{RP}^1$ is by using **inhomogeneous** coordinates. The inhomogeneous coordinate of the point $(x, y)$ is the number $y/x$. Explain why points in $\mathbb{RP}^1$ with homogeneous coordinates $(x, y)$ and $(x', y')$ have the same inhomogeneous coordinates if and only if they represent the same point in $\mathbb{RP}^1$.

**16.** What points of $\mathbb{RP}^1$ can be represented using inhomogeneous coordinates?

**17.** What are the inhomogeneous coordinates of the points on the line $x = 1$?

**18.** Given a point with inhomogeneous coordinates $z$, what is the image of $z$ (in inhomogeneous coordinates) under the map defined by the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$?

**19.** Represent each of the functions $f_k$, $g$, and $h_d$ in matrix form.