Math F200X   Final Exam   Fall 2004

Name: ______________________

Rules:

You have 120 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

No calculators, books, notes, or other aids are permitted.

Turn off anything that might go beep during the exam.

If you need extra space, you can use the back sides of the pages. Please make it obvious when you have done so.

Good luck!

Problem	Possible	Score
1	30	
2	20	
3	10	
4	20	
5	30	
6	20	
7	10	
8	20	
9	20	
10	20	
Total	200	

1. (30 points)

Compute the following derivatives.

a. $\dfrac{d}{dx} 2\sin(x)\cos(x)$

b. $\dfrac{d}{dx} \dfrac{\tan(2x)}{1+x^2}$

c. $\dfrac{d}{dx} \arctan(e^{1+x^3})$

d. $\dfrac{d}{dx} \displaystyle\int_1^{\ln(x)} \frac{1}{1+t^3}\, dt$

2. (20 points)

Let $\mathcal{A}$ be the region of the plane bounded by the y-axis, the line $y = x$, the curve $y = 1 + e^{-x}$, and the line $x = 1$.

a. Make a rough sketch of the region $\mathcal{A}$.

b. Compute the volume of the region $\mathcal{A}$ rotated about the x-axis.

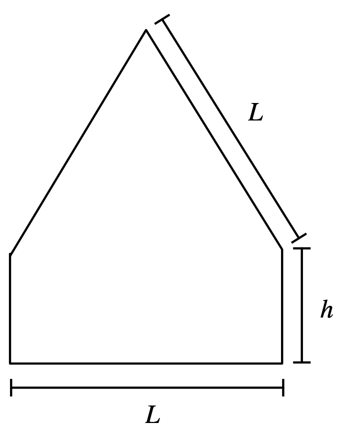
3. (10 points)

Compute the tangent line to the curve

$$ye^y = x$$

at the point $(e, 1)$.

4. (20 points)
Consider the region depicted in the diagram. The top is an equilateral tringle with side length L. The bottom is a rectangle with width L and height h. What values of L and h maximize the area of this region subject to the condition that the perimeter of the region is 1? (*Hint:* The area of an equilateral triangle with side length L is $\frac{\sqrt{3}}{4}L^2$.).



5. (30 points)

Compute the following definite or indefinite integrals.

a. $\int \sqrt{x} - \frac{1}{x} + x^7 \ dx$

b. $\int \tan(x) \sec^2(x) \ dx$

c. $\int_{1/2}^{1} \frac{\sqrt{\ln(2x)}}{x} \ dx$

d. $\int_{1}^{e} \ln(x) \ dx$ *Hint:* $\frac{d}{dx} [x \ln(x) - x] = \ln(x)$.

6. (20 points)

Let $f(x) = x^5 - 3$.

a. Perform two iterations of Newton's method on $f(x)$ starting with an initial value of $x_0 = 1$. You do not need to simplify your final answer.

b. If you continued to perform iterations of Newton's method, what number do you expect your answers would converge to?

7. (10 points)

Water is filling my garage at a rate $r(t) = 5(1 - e^{-t})$ where $r(t)$ is measured in gallons per minute and t is measured in minutes. My garage has no water in it at time $t = 0$. How much water is in my garage three minutes later (i.e. at time $t = 3$)?

8. (20 points)

Sketch the graph of the function

$$f(x) = xe^{1-x^2}.$$

Be sure that your sketch includes any local extreme points, points of inflection, and horizontal and vertical asymptotes.

9. (20 points)

Compute the following limits.

a. $\displaystyle\lim_{x\to\pi} \frac{\sin(x)}{x}$

b. $\displaystyle\lim_{x\to 0} \frac{1-\cos^2(x)}{x^2+1}$

c. $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{2x+1}$

d. $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \left(1+\frac{k}{n}\right)^2 \frac{1}{n}$

10. (20 points)

A moose is standing 9m away from a straight section of the Richardson Highway. Let P be the point on the highway nearest the moose. A car is traveling along the highway towards P at 30m/s. What is the rate of change of the distance between the car and the moose when the car is 100m from point P?