1. Munkres 17.19

2. Munkres 18.4 (See the definition of *imbedding* at the bottom of page 105.)

3. Suppose $\mathcal{T}$ is a topology on $X \times Y$ that satisfies the characteristic property of the product topology. Show that $\mathcal{T}$ **is** the product topology. (This shows that the characteristic property is characteristic.)

4. Munkres 17.13 (Typeset this one.)

5. Munkres 17.14

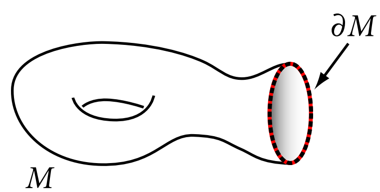
6. A set $A \subseteq X$ is said to be *dense* in X if $\bar{A} = X$.

a) Suppose X is second countable. Prove that it has a countable dense subset.

b) Prove that a metric space is second countable if and only if it has a countable dense subset.

7. Show that the product of an n-manifold N and an m-manifold M is an $n + m$ manifold.

8. An n-dimensional manifold with boundary is a second countable Hausdorff space M such that every point $x \in M$ has an open neighborhood U homeomorphic to an open subset of the upper half space $\mathbb{R}^{n,+} = \{x \in \mathbb{R}^n : x_n \geq 0\}$. Every such homeomorphsim is called a chart. The terminology "manifold with boundary" is historical and mildly misleading: every manifold is a manifold with boundary, but a manifold with boundary typically isn't a manfold! The boundary of M, denoted by ∂M, is the set of points $x \in M$ such that there is a chart taking x to a point of $\partial\mathbb{R}^{n,+}$. It can be shown (you are not required to show this) that if one chart takes a point x to a point of $\partial\mathbb{R}^{n,+}$, then *every* chart does.



A manifold with boundary.

This notion of boundary is not the same as the topological notion of boundary we gave in class. Since the topological boundary of M is the empty set (and therefore not particularly interesting), there is usually no confusion in using the symbol ∂M to mean something different for manifolds with boundary.

Anyway, enough my rambling. Here's the problem:

Let M be an n-dimensional manifold with boundary. Show that ∂M is an $(n-1)$-dimensional manifold.