Math F608: Homework 9 **Due: November 13**

1. Evans 3.5.3

2. Evans 3.5.14

3. Consider the traffic flow problem

$$\begin{aligned} u_t + F(u)_x &= 0 \\ u(x,0) &= h(x) \end{aligned}$$

where u is the traffic density and $F(u)$ is the flux function given by

$$F(u) = cu(1 - u/u_{\max}),$$

where c is the free speed of the road and $u_{\max}$ the maximum density of cars on the road.

a. Find the solution of the PDE with initial data

$$h(x) = \begin{cases} \frac{u_{\max}}{2} & \text{for } x < 0 \\ u_{\max} & \text{for } x \geq 0. \end{cases}$$

Be sure to describe the shock that develops.

b. Find the solution of the PDE with initial data

$$g(x) = \begin{cases} u_{\max} & \text{for } x < 0 \\ 0 & \text{for } x \geq 0. \end{cases}$$

4. Solve the Cauchy problem

$$\begin{aligned} u_x^2 + u_y &= 0 \\ u(x,0) &= x. \end{aligned}$$

5. Consider the Eikonal equation $u_x^2 + u_y^2 = n(x,y)^2$ where $n(x,y) = x$ in the region of the plane where $x > 0$. Find a solution of the Eikonal equation with $u(x,0) = 0$.