Math F608: Homework 8 **Due: October 30**

1. Evans 2.5.18

2. Consider Evans 2.5.18 again in the two dimensional setting. Show that

$$u(x,t) \le Ct^{-1/2}.$$

3. Let U be a bounded smooth domain. Show there is at most one solution of

$$\begin{aligned} u_{tt} - \Delta u &= 0 \quad \text{in } U \times (0,T) \\ u(x,0) &= g(x) \\ u_t(x,0) &= h(x) \\ u(x,t) &= 0 \quad \text{for } x \in \partial U. \end{aligned}$$

Hint: Prove an energy equality.

4. Let Q be the square $[0,1] \times [0,1]$. Use separation of variables to find a formal solution of the problem

$$\begin{aligned} u_{tt} - \Delta u &= 0 \quad \text{in } Q \times (0,T) \\ u(x,0) &= g(x) \\ u_t(x,0) &= h(x) \\ u(x,t) &= 0 \quad \text{for } x \in \partial Q. \end{aligned}$$

5. A function $u : \mathbb{R}^n \times \mathbb{R}$ is called a plane wave if $u(x,t) = F(a \cdot x - t)$ for some fixed vector $a \in \mathbb{R}^n$.

a. Find a condition on the vector a that ensures a plane wave is a solution of the wave equation $u_{tt} - \Delta u = 0$.

b. Determine a necessary and sufficient condition on initial data $g(x)$ and $h(x)$ to ensure the corresponding solution of the wave equation is a plane wave.