1.

a. Find a solution u of the wave equation on the half line with initial displacement xe^{-x}. That is, solve

$$\begin{aligned} u_{tt} - u_{xx} &= 0 \quad \text{on } (0,\infty)\times(0,\infty) \\ u(x,0) &= xe^{-x} \\ u_t(x,0) &= 0 \\ u(0,t) &= 0. \end{aligned}$$

Notice that even though the initial data are smooth, the solution is **not** a classical solution. Determine where the singularities of the solution are located.

b. For the problem

$$\begin{aligned} u_{tt} - u_{xx} &= 0 \quad \text{on } (0,\infty)\times(0,\infty) \\ u(x,0) &= g(x) \\ u_t(x,0) &= h(x) \\ u(0,t) &= 0. \end{aligned}$$

find necessary and sufficient conditions on g and h to ensure u is a classical (i.e. C^2) solution.

2. Evans 2.5.16

3. Evans 2.5.17

4. Find an exact solution of the Darboux problem

$$\begin{aligned} u_{tt} - u_{xx} &= 0 \quad \text{for } t > |x| \\ u(x,x) &= \phi(x) \quad \text{for } x \geq 0 \\ u(-x,x) &= \psi(x) \quad \text{for } x \geq 0 \end{aligned}$$

where ϕ and ψ are C^2 functions on $[0,\infty)$ that satisfy $\phi(0) = \psi(0)$.

5. Find a solution u of the wave equation on the half line with a forcing term on the left boundary.

$$\begin{aligned} u_{tt} - u_{xx} &= 0 \quad \text{on } (0, infty)\times(0,\infty) \\ u(x,0) &= g(x) \\ u_t(x,0) &= h(x) \\ u(0,t) &= \alpha(t) \end{aligned}$$

where $\alpha(0) = 0$. Determine conditions on α that will guarantee the solution is C^2.