Math F608: Homework 5 **Due: October 9**

1. Evans 2.5.12

2. Evans 2.5.13

3. Prove the follwing weak maximum principle for the heat equation in $\mathbb{R}^n \times (0,T)$. Let u be bounded and continuous on $\mathbb{R}^n \times [0,T]$ and be a $C^{2,1}(\mathbb{R}^n \times (0,T)$ function satisfying

$$u_t - \Delta u \leq 0$$

on $\mathbb{R}^n \times (0,T)$. Show that

$$\sup_{\mathbb{R}^n \times [0,T]} u(x,t) \leq \sup_{\mathbb{R}^n} u(x,0).$$

Hint: One solution to this problem takes advantage of the fact that $v(x,t) = \frac{1}{2}|x|^2 - t$ solves the heat equation. Consider functions of the form $u_\epsilon = u - \epsilon v$.

4.

a. Find a solution to the system of ODEs

$$u'(t) + v(t) = 0$$
$$v'(t) - u(t) = 0$$

with initial condition $u(0) = a$ and $v(0) = 0$. Even if you haven't see systems of ODEs before, you should be able to find a solution with a little playing around.

b. Use Duhamel's principle to solve the following non-homogeneous ODE

$$u'(t) + v(t) = f(t)$$
$$v'(t) - u(t) = 0$$

with initial conditions $u(0) = v(0) = 0$.