Math F608: Homework 2 **Due: September 18**

1. Consider the initial value problem

$$\begin{aligned} u_t + x^2 u_x &= 0 \quad \text{in } \mathbb{R}\times(0,\infty) \\ u &= g \quad \text{on } \mathbb{R}\times\{0\}. \end{aligned}$$

Compute the characteristic curves of this PDE. Carefully consider the region of the plane that are covered by characteristic curves. Then answer (with justification) the following questions. Given a C^1 function g, is there a solution of the initial value problem? If solutions exist, are they unique?

2. Let U be the square domain $\mathbb{R}\times(0,\infty)$. Suppose u is a solution of the initial value problem

$$\begin{aligned} u_{tt} + u_{xx} &= 0 \quad \text{in } U \\ u(x,0) &= f(x) \\ u_t(x,0) &= g(x). \end{aligned}$$

Compute the following in terms of the functions f and g:

$$u_{xt}(0,0)$$

and

$$u_{tttx}(0,0).$$

Assume that u has derivatives up to fourth order. Compute the power series, up to fourth order terms, for u at the point $(0,0)$. This exercise hints at how one might come up with a power series solution for a PDE given initial data. The theorem you might be interested in looking up at this point in the Cauchy-Kowaleski theorem.

3. Evans 2.5.2

4. Evans 2.5.3

5. Find a fundamental solution for the differential equation

$$L = \frac{d}{dx} - a$$

on $\mathbb{R}$. (I.e. a solution of $Lu = \delta$.)