1. Compute the Fourier transform of the following functions.

   a. $f(x) = \dfrac{x}{|x|}$.

   b. $f(x) = \begin{cases} 1 & 0 < x < a \\ 0 & \text{otherwise} \end{cases}$.

   c. $f(x) = e^{-|x|}$

   d. $H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$.

   e. $f(x) = x$

   f. $f(x) = \sin(\omega x)$

2. Let f be a continuous function such that $\hat{f}$ satisfies $\hat{f}(\xi) = 0$ for $|\xi| > \pi$. Show that

$$f(x) = \sum_{k=-\infty}^{\infty} f(k) \frac{\sin(\pi(x-k)}{\pi x - k}$$

3. Show that if F is a tempered distribution then $\widehat{\partial F} = 2\pi i \xi \hat{F}$

4. Use the Fourier transform to solve the problem

$$u_t = u_{xx} + u_x$$

on the space $\mathbb{R} \times (0, \infty)$ with initial condition $u(x, 0) = g(x)$.

5. Use the Fourier transform to solve the problem

$$u_{xx} + u_{yy} = 0$$

on the infinite strip $\mathbb{R} \times (0, 1)$ with boundary data $u(x, 0) = 0$ and $u(x, 1) = f(x)$.