Math F608: Homework 1 **Due: September 8**

1. Let $u(x,y,z) = (x^2, y^2, z^2)$, and let U be the cylinder with $x^2 + y^2 < 4$ and $0 < z < 2$. Compute

$$\int_{\partial U} u \cdot \nu \, dS$$

directly and then also indirectly using the Divergence Theorem. (Do not do any hard integrals for this problem! Use symmetry and geometric arguments wherever possible.)

2. Let f be a smooth function and let $u(t) = \int_0^t f(xt)\; dx$. Show that u sastisfies the ordinary differential equation

$$tu' + u = 2tf(t^2).$$

3. Find the solution of the ordinary differential equation

$$\begin{aligned} u' + cu &= 0 \\ u(0) &= a \end{aligned}$$

4. (Evans 2.5.1) Write down an explicit formula for a function u solving the initial-value problem

$$\begin{aligned} u_t + b \cdot Du + cu &= 0 \quad \text{in } \mathbb{R}^n \times (0,\infty) \\ u &= g \quad \text{on } \mathbb{R}^n \times \{t = 0\}. \end{aligned}$$

5. Evans 1.5.1 (odd numbered PDEs only)